A Logic Puzzle Regarding Common Knowledge

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Twenty three blue-eyed logicians and thirteen brown-eyed logicians are in a room. At the end of each day, those logicians who know for sure the color of their own eyes leave the room. Each logician can see the eyes of everyone else but not his own eyes. One day, a public announcement is made that "at least one logician in the room has blue eyes".

(i) How many days pass after the announcement before any logicians leave the room?
23 days.

Justification:

- The announcement makes it common knowledge that at least 1 logician has blue eyes

- If a person saw no logicians with blue eyes, he would know his own eyes are blue. People that don't leave the room on the 1st night make it common knowledge that they *see* at least **1** logician with blue eyes

- If a person, say Bob, saw only 1 logician, say Jack with blue eyes, and Jack did not leave the room on the first day, then Bob would know that his eyes are blue (why?) and would leave on the second night. Hence, people that don't leave the room on the 2nd night make it common knowledge that they *see* at least **2** logicians with blue eyes.

- Continuing this argument, people that don't leave the room on the 22nd night make it common knowledge that they *see* at least **22** logicians with blue eyes.

- At this point, each logician with blue eyes can infer that he is one of the blue-eyed logicians that other blue-eyed logicians see. Hence, all blue-eyed logicians leave the room on the 23rd night.

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(*ii*) How many days pass after the announcement until every logician has left? 24 days.

Justification: When the blue-eyed logicians leave the room, it is because each of them could see *exactly* 22 blue-eyed logicians and knew that other blue-eyed logicians could also see 22 blue-eyed logicians. Their departure made it common knowledge that there were no more blue-eyed logicians in the room. Hence, all brown-eyed logicians leave on the next night.