Social Welfare

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Western

social welfare functions

How to choose a **public policy**, that affects **different individuals** with (typically) **different preferences** over policies?

examples

- Harmonized Sales Tax rate
- Free trade agreements
- Ticket sales
- Display of news on social media
- Net neutrality
- Roads or bike lanes
- Ontario Hydro
- Consumption and production

A social choice problem consists of

- A set $\mathcal{A}$ of alternatives A
- A set of individuals i
- For each individual i, a preference ranking $\succ_i$ over alternatives

We are after a social ranking $\succ^*$ over alternatives

- Principle for deciding which outcomes are "good for society"
- Should depend on the preferences of the individuals

A **social welfare function** is a mathematical function that takes as input a list of preferences $(\succ_1, \succ_2, \ldots, \succ_n)$ and produces as output a single preference ranking $\succ^*$

- Majority rule with two alternatives and an odd number of individuals
- Sequential plurality (top choice for most individuals) ▷
- Condorcet criterion (winners of pairwise elections) ▷
- Borda criterion (point-system voting) ▷
- Utilitarian (maximize sum of utilities)
- Rawlsian (maximize the utility of the worst-off individual)

arrow's theorem

universal domain

What are some minimal properties a SWF should satisfy?

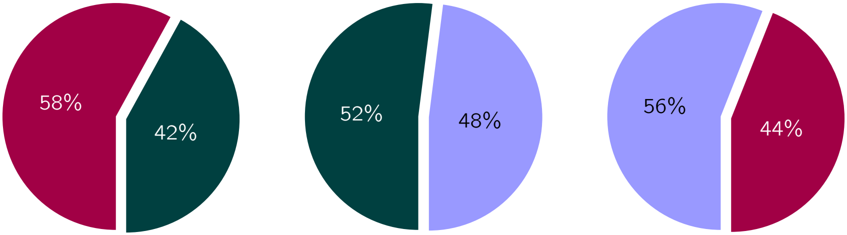
A SWF satisfies **universal domain** (UD) if every possible preference list results in a well defined social-ranking output

- The Condorcet criterion fails UD
- Consider the following example

1	2	3
A	B	C
B	C	A
C	A	B

- According to the Condorcet criterion, $B \succ^* C$, $C \succ^* A$, and $A \succ^* B$
- How do we choose an alternative from $\mathcal{A}$?

Condorcet cycle



May deal $\succ^*$ no deal no deal $\succ^*$ remain remain $\succ^*$ May deal

Thinking about your view of Brexit, for each of the following please say if it would be your first preference, second preference, or third preference.

What are some minimal properties a SWF should satisfy?

A SWF satisfies **unanimity** (U) if, whenever it happens that for some pair of alternatives A and B, every individual i ranks $A \succ_i B$, the corresponding social ranking also ranks $A \succ^* B$

independence of irrelevant alternatives

What are some minimal properties a SWF should satisfy?

A SWF satisfies **independence of irrelevant alternatives** (IIA) if, if the social ranking of A versus B depends only on the individuals' rankings of those two alternatives

- The sequential plurality rule fails IIA
- Consider the following example

1	2	3	4	5	6	7	8	9
A	A	A	A	B	B	B	B	B
B	B	B	B	C	C	C	A	A
C	C	C	C	A	A	A	C	C

- According to the sequential plurality rule, $B \succ^* A$

- The sequential plurality rule fails IIA
- If the preference of individuals 8 and 9 changes as follows

1	2	3	4	5	6	7	8	9
A	A	A	A	B	B	B	C	C
B	B	B	B	C	C	C	B	B
C	C	C	C	A	A	A	A	A

- Now, $A \succ^* B$

minimal properties a SWF should satisfy

- We have argued a good SWF should *at least* satisfy UD, U, and IIA
- These are minimal requirements
- They say nothing about equity, fairness, or how to conciliate conflict
- A good SWF should satisfy these, and probably some more requirements
- Is there any such SWF?

A SWF is a **dictatorship** if there exists some individual i such that the social raking $\succ^*$ is always exactly the same as $\succ_i$, regardless of the preferences of other individuals

Dictatorships satisfy our minimal requirements

- UD because there is always an answer ($\succ^* = \succ_i$)
- U because if A is unanimously better to B, then $A \succ_i B$, and thus $A \succ^* B$
- IIA because the social ranking of alternatives A and B only depends on the dictator's individual ranking of A and B

what else satisfies requirements?

- Simple majority fails UD
- Condorcet criterion fails UD
- Sequential plurality fails IIA
- Borda rule? (homework)

Arrow's impossibility theorem — If a SWF satisfies U, UD, and IIA, then it is dictatorial

what do we do now?

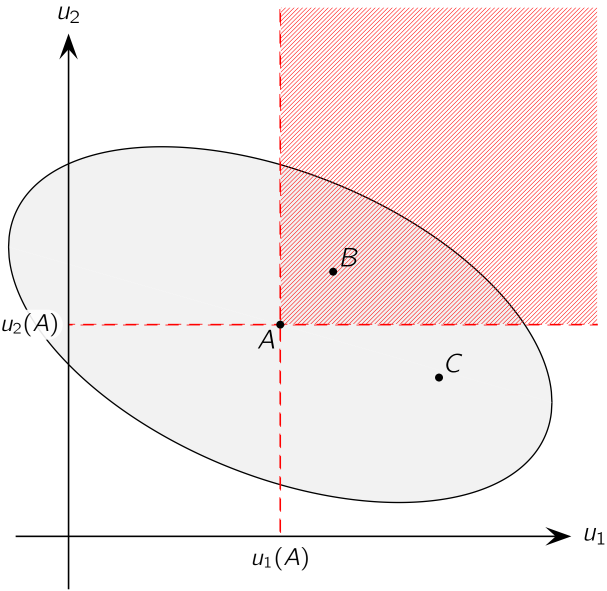
- Relax some of the “minimal” requirements? Which?
  - U is an important requirement we would not want to drop
  - IIA? Maybe. . .
  - Restricted domains? Yes, in this course
- More information? If we could measure utility we could use
  - Utilitarian (maximize sum of utilities) ▷
  - Rawlsian (maximize the utility of the worst-off individual) ▷
- Unfortunately we cannot measure utility in general domains

unanimity and efficiency

Pareto dominance

Alternative A **Pareto dominates** alternative B if **every** individual prefers A to B, i.e., $A \succ_i B$ for every individual i

- Pareto dominance is a SWF designed around unanimity
- It satisfies U and IIA, but it fails UD
- In many cases, it yields incomplete rankings
  - Who gets the last ticket?
  - Public school assignment
  - Introducing Uber

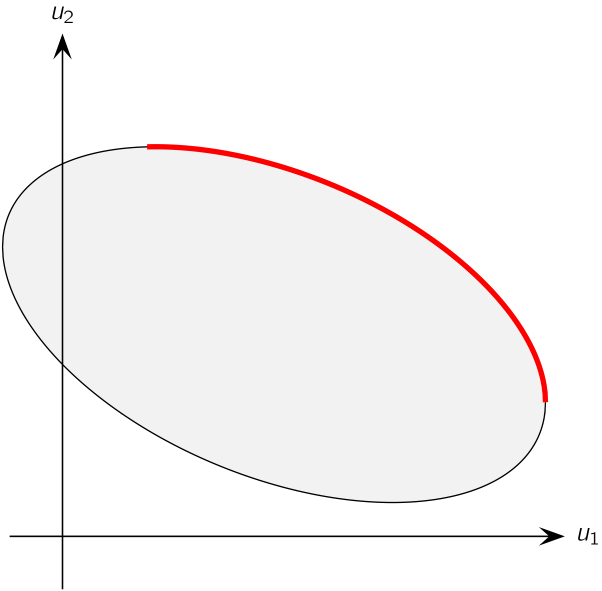


alternative A is Pareto dominated by B but not by C

Pareto efficiency

An alternative A is **Pareto efficient** if there is no other alternative that Pareto dominates it

- Compelling prescription — should not choose any alternative which is Pareto dominated, when it is feasible to choose an alternative that Pareto dominates it
- Fundamental principle of economics, often misused
- Not every Pareto efficient alternative dominates every alternative which is not Pareto efficient
- Better to think in terms of Pareto improvements



the set of Pareto efficient alternatives corresponds to the **Pareto Frontier**

willingness to pay

How large do we have to make the pile before you take the money?

Pareto efficiency with money

- Suppose there is one ticket and two people without tickets left
- Anna's willingness to pay is \$200
- Bob's willingness to pay is \$100
- What are the implications of Pareto efficiency?
- Give the ticket to the individual with the highest willingness to pay

restricted domain

monetary transfers

- Suppose monetary transfers are possible and can be enforced
- A monetary transfer scheme can be represented by numbers $t_1, t_2, \ldots, t_n$
  - t_i represents the amount paid by individual i (could be negative)
  - $\sum_i t_i$ is the total surplus (or deficit)
  - $\sum_i t_i = 0$ means that the scheme is budget balanced

- Restricted domain of preferences that can be represented as follows
- Individual i's value for alternative A is $v_i(A)$
- Individual i's utility for alternative A and transfer t_i is

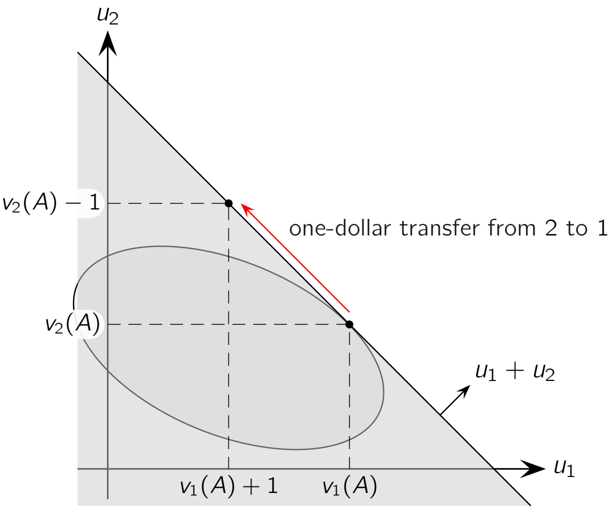
$$u_i(a, t_i) = v_i(a) - t_i$$

- The difference $v_i(a) - v_i(b)$ captures i's willingness to pay for having alternative A instead of alternative B
- How restrictive is this domain?

If transfers are possible and all agents have quasilinear preferences, then (A, t) is Pareto efficient if and only if

$$\sum_i v_i(A) \geq \sum_i v_i(B)$$

for every other alternative B in $\mathcal{A}$



now the Pareto frontier is a line with slope -1 specific SWFs

appendix

- The alternative with the most “top choice votes” is at the top of the social ranking
- Remove that alternative from the individual rankings, leaving the rest intact
- With the new individual rankings, find the alternative among those that remain with the most “top choice votes”
- That alternative is places second in the social ranking
- Continue until all alternatives are ranked

sequential plurality example

1	2	3	4	5
A	A	B	B	B
B	C	A	A	C
C	B	C	C	A

- B has the most "top choice votes"
- Thus $B \succ^* A$ and $B \succ^* C$
- Once B is removed, A has more "top choice votes" than C
- Thus $A \succ^* C$

Condorcet criterion

- For each pair of alternatives A and B, count how many individuals prefer A to B and vice versa
- If more individuals prefer A to B, then A is socially preferred to B
- For the following example, following the Condorcet criterion yields $A \sim^* B$, $B \succ^* C$, and $A \succ^* C$

1	2	3	4
A	A	B	B
B	C	A	A
C	B	C	C

Condorcet vs. plurality

1	2	3	4	5	6	7
L	L	L	C	C	R	R
C	C	C	L	R	C	C
R	R	R	R	L	L	L

- Plurality rule — $L \succ^* C \succ^* R$
- Condorcet criterion — $C \succ^* L \succ^* R$

Borda criterion

- Suppose there are n alternatives
- For each individual i assign points to alternatives as follows
  - i's most preferred alternative gets n points
  - i's second most preferred alternative gets $n - 1$

    $\vdots$

  - i's least preferred alternative gets 1 point
- Rank alternatives according to the total number of points assigned to them

Borda criterion example

1	2	3	4
A	A	B	B
B	C	A	A
C	B	C	C

preferences

	1	2	3	4	**total**
A	3	3	2	2	**10**
B	2	1	3	3	**9**
C	1	2	1	1	**5**

points assigned

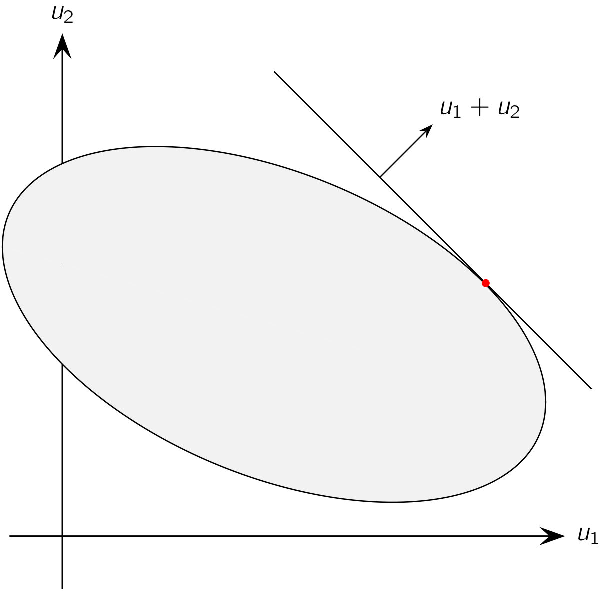
mill and rawls

appendix

- Suppose that we can measure utility
- For each individual i we have a utility function u_i over alternatives
- Utilitarianism says alternative A is socially preferred to alternative B if it generates more total utility for society

$$\sum_i u_i(A) > \sum_i u_i(B)$$

- Satisfies UD, IIA and U
- Susceptible to changes of scale (depends on cardinal information)
- Assumes same scale can be used to compare utility across individuals

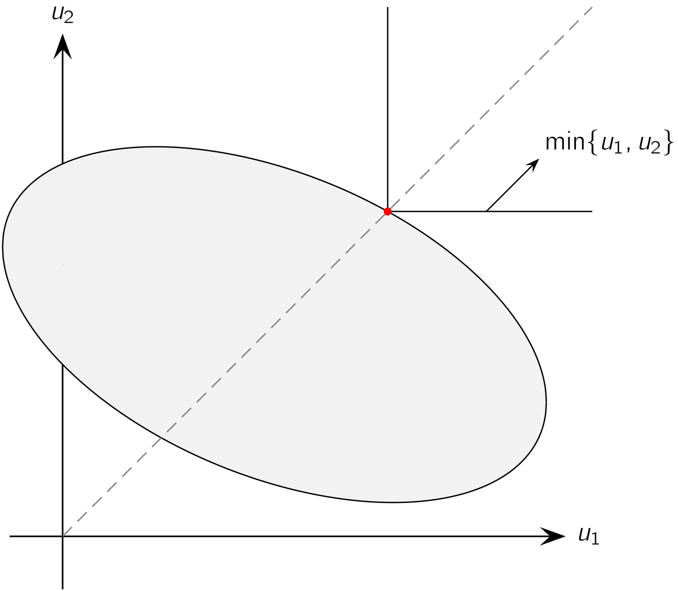


mill – people are treated like perfect substitutes

- Suppose that we can measure utility
- For each individual i we have a utility function u_i over alternatives
- Rawls says alternative A is socially preferred to alternative B if the worse off individual under A is better off than the worse off individual under B

$$\min_i u_i(A) > \min_i u_i(B)$$

- Veil of ignorance — what would individuals prefer before they knew their place in society?
- Satisfies UD, IIA and U
- Susceptible to changes of scale (depends on cardinal information)
- Assumes same scale can be used to compare utility across individuals



rawls – people are treated like perfect complements