Competitive Markets

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Economics 2261 · Intermediate Micro II · Winter 2020

competitive markets

consumers
price takers
endowment/income
choose consumption
maximize utility

firms
price takers
technology/cost function
choose production
maximize profits

- Subindices
  - k for products
  - i for consumers
  - j for firms
- Notation for vectors
  - Bold font on slides and print
  - "Bar" accent on the blackboard (e.g., $\bar{x}$)
- Aggregate vs. individual variables
  - Uppercase for aggregate quantities
  - Lowercase for individual quantities
- Examples
  - Consumer i's consumption bundle is $\mathbf{x}_i = (x_{i1}, \ldots, x_{in})$
  - The amount of product k produced by firm j is y_{jk}
  - The market demand for product k is $D_k = \sum_i d_{ik}$

market supply

- Firm j chooses the output of its single-product to maximize profits

$$\max_{y \geq 0} \qquad py - C(y)$$

- The solution $s_j(p)$ is called firm j's individual supply
- The market supply specifies the total quantity supplied

$$S(\mathbf{p}) = \sum_j s_j(\mathbf{p})$$

- The first order condition for this problem is

$$\mathrm{MC}(y^*) = p$$

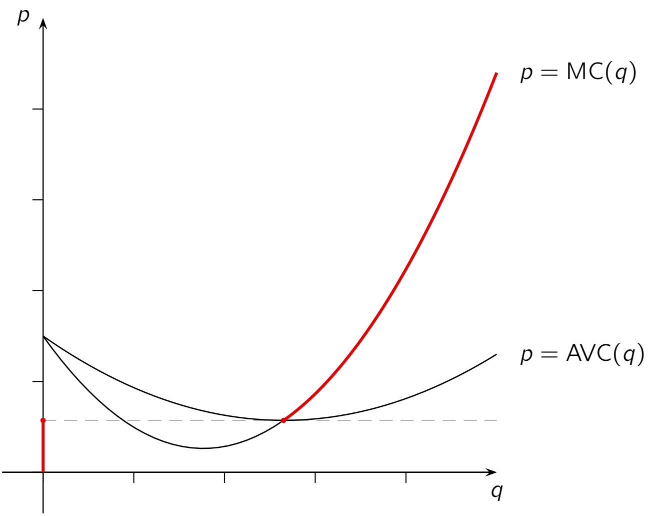
- The firm also has the option of not producing anything at a cost $C(0)$, which could be positive in the short run (why?)

- Producing y^* instead of 0 is profitable only when

$$py^* - C(y^*) \geq -C(0)$$

- Rearranging terms, the price must be greater than the average variable cost

$$\mathrm{AVC}(y^*) := \frac{C(y^*) - C(0)}{y^*} \leq p$$



supply coincides with the MC curve above the AVC curve

A firm's **producer surplus** measures how much the firm benefits from having access to the market

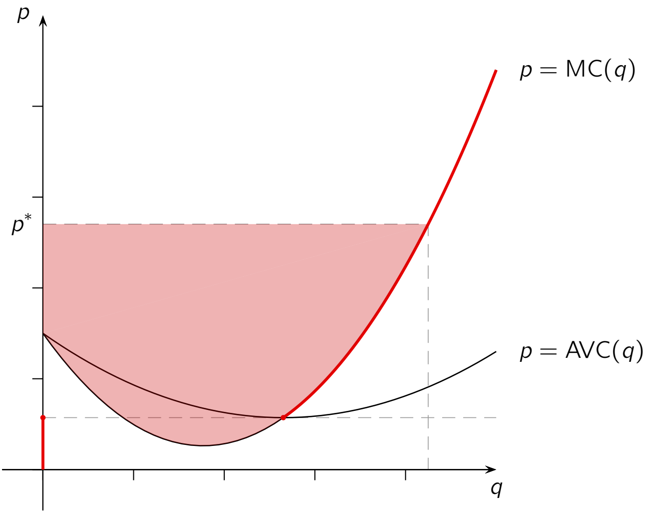
$$py^* - \Big(C(y^*) - C(0)\Big)$$

- Fundamental theorem of calculus

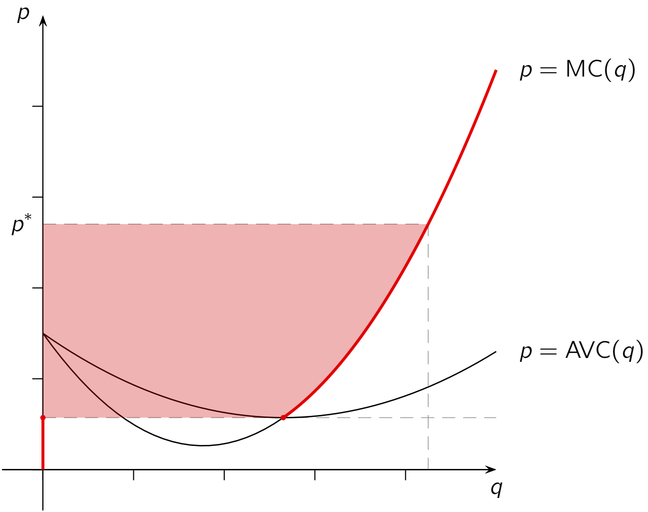
$$C(y^*) - C(0) = \int_0^{y^*} \mathrm{MC}(y)\, dy$$

- Firm surplus is thus given by

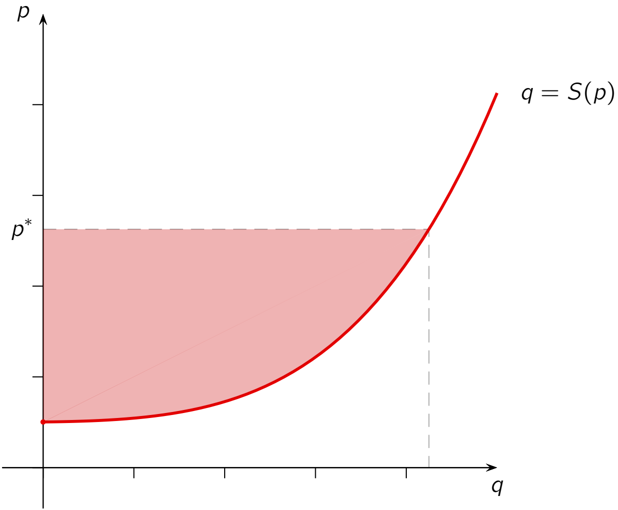
$$py^* - \int_0^{y^*} \mathrm{MC}(y)\, dy$$



firm j's surplus equals the area over j's MC curve and below p^*



firm j's surplus also equals the area over j's supply curve and below p^*



the industry surplus for a product equals
the area under the market supply curve and above p^*

short-run profits

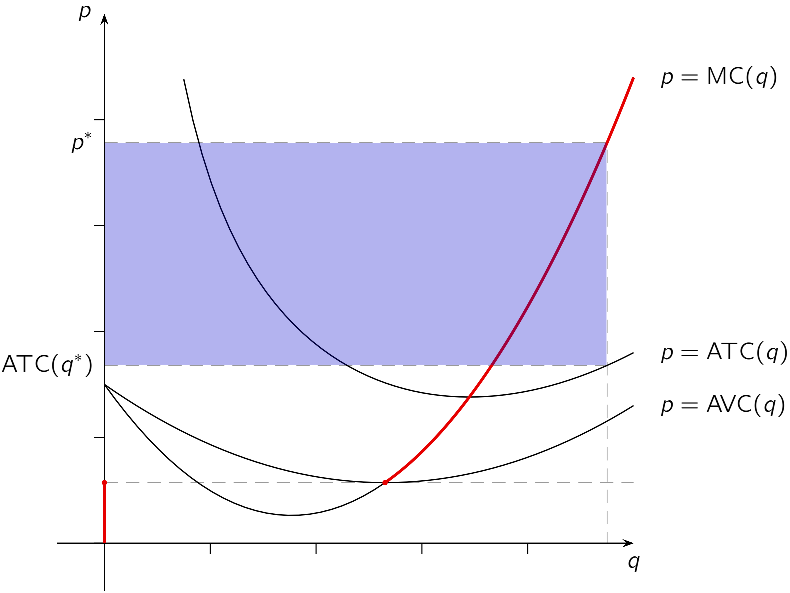
- The profits of firm j equal revenue minus cost

$$\pi = py - C(y)$$

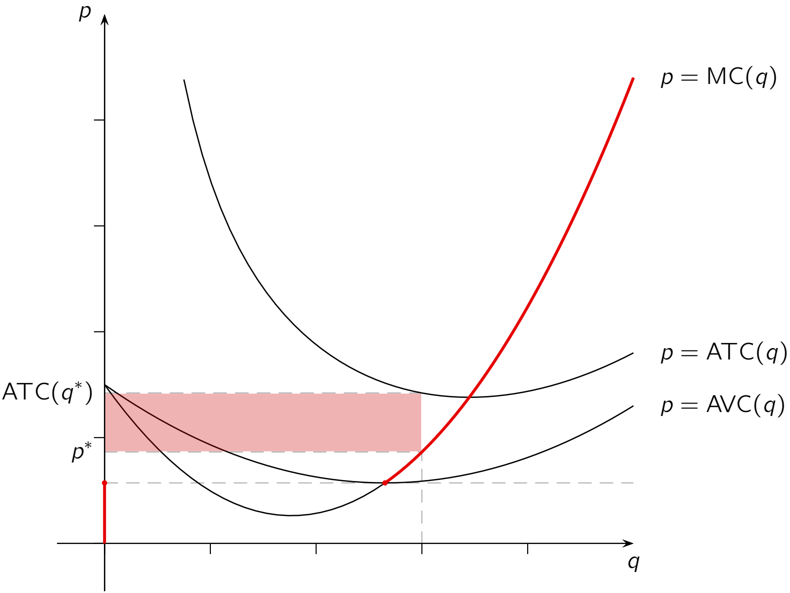
- Expressed in terms of average total costs as

$$\pi = y\left(p - \frac{C(y)}{y}\right) = y\big(p - \mathrm{ATC}(y)\big)$$

- In the short run, profits can be positive or negative (if there are fixed costs)
- Positive if price exceeds average cost, negative otherwise



profits are positive when the market price exceeds the average total cost



profits are negative when the market price exceeds the average total cost

market demand

- Consumer i chooses the consumption that solves:

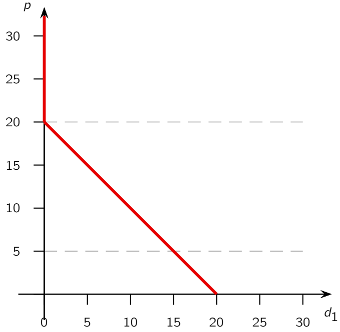
$$\max_{x_{i1},\dots,x_{in}} \quad u_i(\mathbf{x}_i)$$
$$\text{st} \quad \sum_k p_k x_{ik} = m_i$$

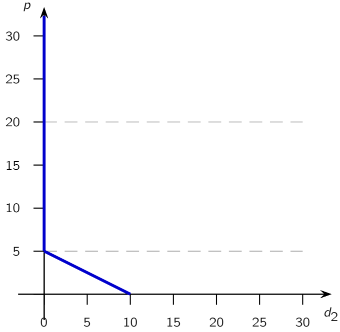
- The solution is called consumer i's individual demand

$$\mathbf{d}_i(\mathbf{p}, m_i) = \big(d_{i1}(\mathbf{p}, m_i), d_{i2}(\mathbf{p}, m_i), \dots, d_{in}(\mathbf{p}, m_i)\big)$$

- The market demand specifies the total quantity demanded

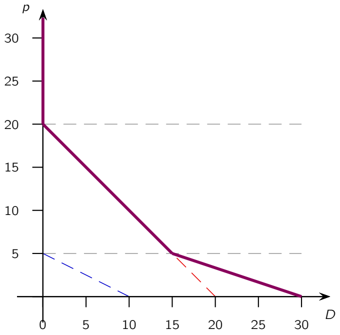
$$\mathbf{D}(\mathbf{p}, \mathbf{m}) = \sum_i \mathbf{d}_i(\mathbf{p}, m_i)$$





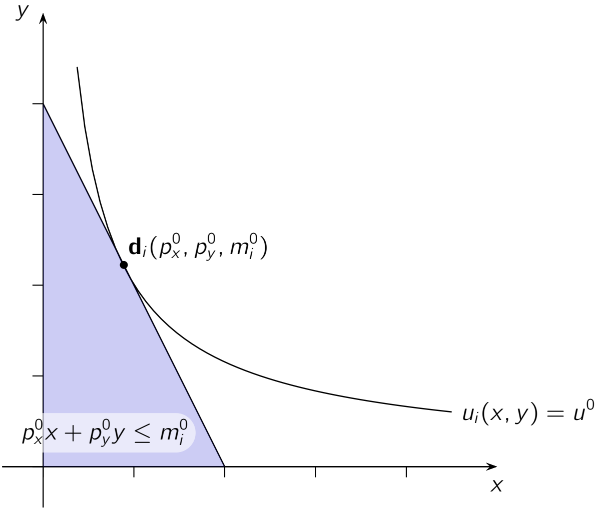
$$d_1(p) = \begin{cases} 20 - p & \text{if } p \leq 20 \\ 0 & \text{if } p > 20 \end{cases}$$

$$d_2(p) = \begin{cases} 10 - 2p & \text{if } p \leq 5 \\ 0 & \text{if } p > 5 \end{cases}$$

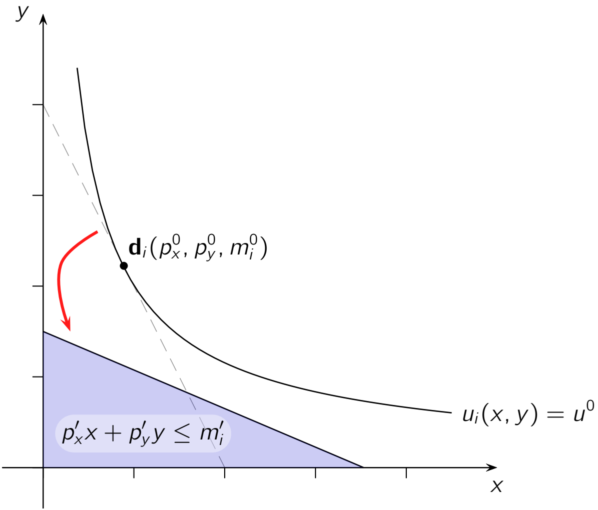


$$D(p) = d_1(p) + d_2(p) = \begin{cases} 30 - 3p & \text{if } p \leq 5 \\ 20 - p & \text{if } 5 \leq p \leq 20 \\ 0 & \text{if } p > 20 \end{cases}$$

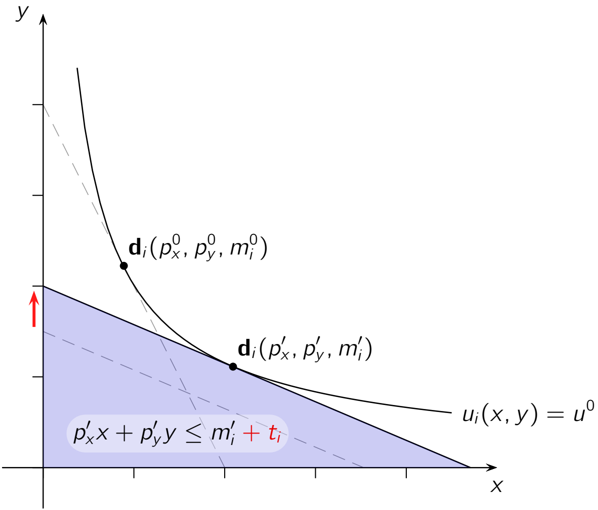
- How to evaluate public policies in terms of welfare?
- Pareto dominance is great but will only take us so far
- Can we measure willingness to pay?



u^0 – level of utility the consumer can afford before policy



suppose policy changes prices to (p'_x, p'_y) and i's wealth to m'_i,
making consumer i worse off



compensating variation – transfer that would make i
be able to exactly afford previous utility level under new prices

- Compensating variations measure of the effect of a policy on consumer welfare in monetary units
- Same units make it possible (?) to aggregate across consumers
- Kaldor Hicks criterion – evaluate policy from the sign of the sum of compensating variations

$$\sum_i t_i$$

caveats

- Compensations are hypothetical, otherwise relative prices would change

- Pareto improvement $\Rightarrow$ sum of compensations is positive
- But sum of compensations is positive $\not\Rightarrow$ Pareto improvement

- Measures average effect ignoring effects on distribution/inequality
- Maybe consider different ways to aggregate

- Difficult to compute, requires lots of information

- Suppose the share of income spent on good k is small
- i's demand for k can be approximated by solving the problem

$$\begin{aligned} \max_{x_{ik}} \quad & v(x_{ik}) + y \\ \text{st} \quad & p_k x_{ik} + y = m_i \end{aligned}$$

- y – money that i reserves to purchase goods other than k
- With quasilinear preferences, i's demand for k takes the form

$$v'(d) = p_k$$

Consumer surplus measures the change in utility from having access to a market

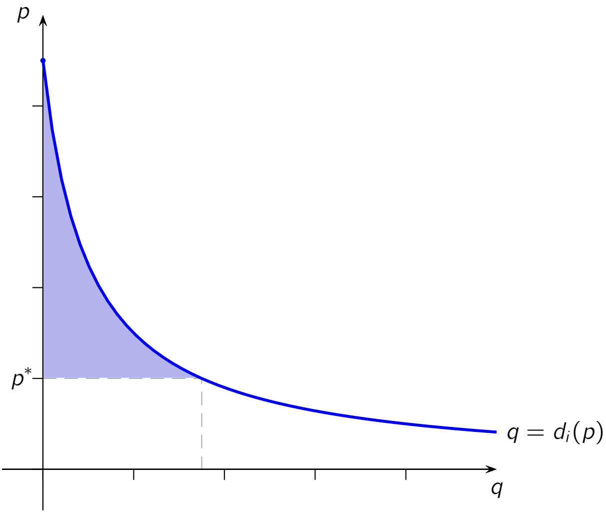
- With quasilinear preferences, consumer surplus equals

$$\big(v(x^*) - v(0)\big) - px^*$$

- From the fundamental theorem of calculus, consumer surplus equals

$$\int_0^{x^*} v'(x)\,dx - px^*$$

- With quasilinear preferences, compensating variations equal changes in consumer surplus



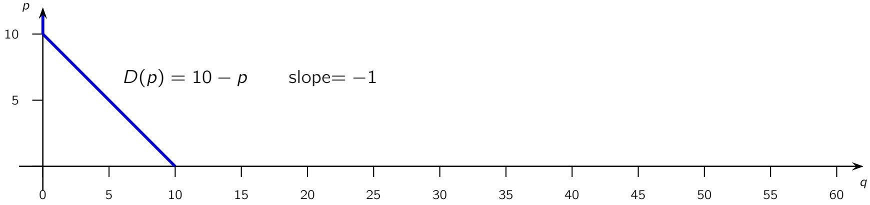
consumer i's consumer surplus equals the area under his/her demand and above p^*

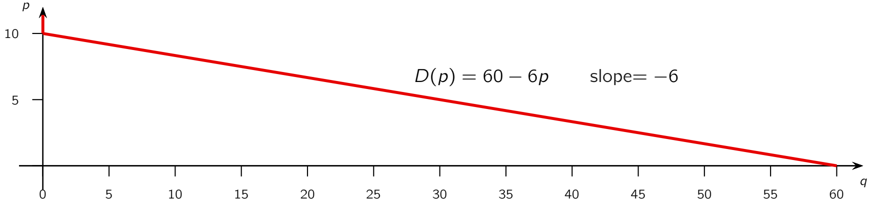
consumer welfare – summary

1. Pareto criterion
   - Ideal way to evaluate policies, when it is informative
   - Often it is not informative
2. Hicks criterion
   - Consistent with Pareto, but more informative
   - Makes cross-agent utility comparisons in monetary units
   - Requires taking a stance on mean vs. variance
3. Consumer surplus (Marshall)
   - Easy to compute
   - Coincides with Hicks under quasilinear preferences

elasticity

How sensitive are demand and supply to changes in prices?





Same demand, different units!

The derivative (slope) is a bad measure of sensitivity for our purposes

- The derivative of y with respect to y measures how y changes in response to infinitesimal changes of x

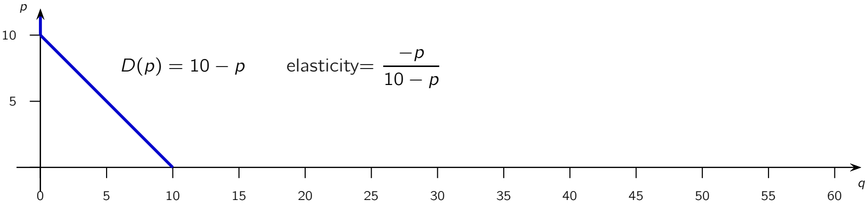
$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

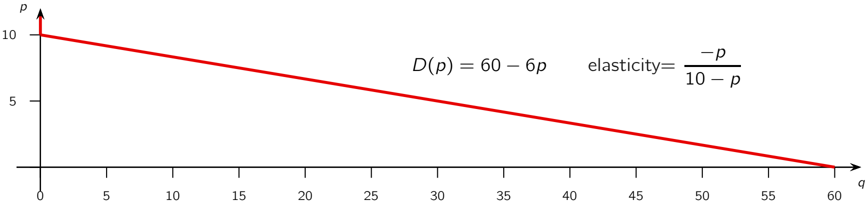
- Percentage changes $\Delta\% x = \Delta x / x$ do not depend on units

- The **elasticity** of y with respect to y measures the percentage change of y in response to infinitesimal percentage changes of x

$$\varepsilon_{y,x} = \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} = \frac{x}{y} \frac{dy}{dx}$$

- Elasticities play an important role to determine revenue, welfare effects, tax incidence



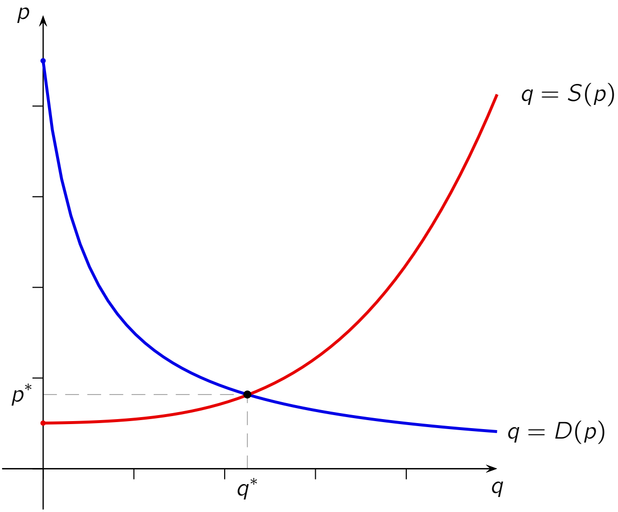


Elasticity is units-free

partial equilibrium

- We know how consumers and firms behave in a competitive environment, as a function of prices
- How are prices determined?
- One idea: prices adjust until markets clear

A **partial equilibrium** for product k consists of a price p^* and quantity q^* such that $D_k(p^*) = S_k(p^*) = q^*$



partial equilibria correspond to intersections of the supply and demand curves

- **Law of demand** – demand is increasing in prices
  - Excludes Giffen goods (junk food, public transportation)
  - Excludes Veblen goods (status goods, low-quality expensive clothes)
- **Law of supply** – supply is decreasing in prices and $S(0) = 0$
  - Excludes industries operating with decreasing marginal costs
- Both demand and supply are continuous functions

Under these assumptions, there always exist a unique partial equilibrium

- Suppose that demand and supply for a product are given by

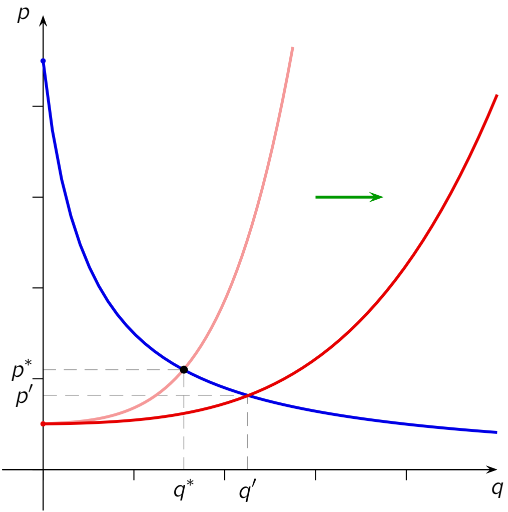
$$D(p) = \frac{12}{p-1} \qquad S(p) = 2p$$

- Then we can find the (unique) partial equilibrium price as follows

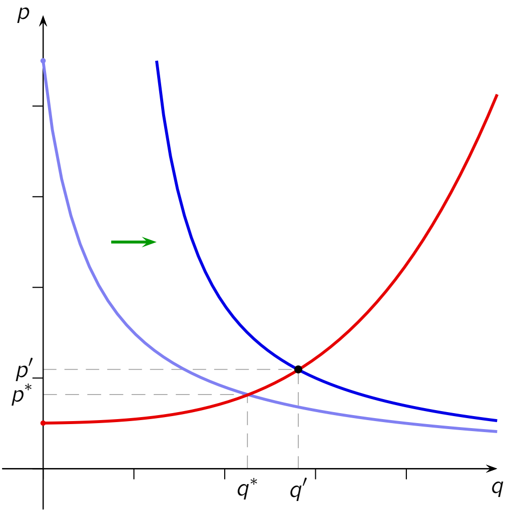
$$\begin{aligned} \frac{12}{p^*-1} = 2p^* \quad &\Leftrightarrow \quad 12 = 2p^{*2} - 2p^* \\ &\Leftrightarrow \quad p^{*2} - p^* - 6 = 0 \\ &\Leftrightarrow \quad (p^* - 3)(p^* + 2) = 0 \end{aligned}$$

- For markets to clear we need $p^* > 0$
- Hence, we can conclude $p^* = 3$ and $q^* = 6$

How do the equilibrium outcomes react to changes in demand or supply?



increase in supply → lower prices and higher output



increase in demand → higher prices and higher output long run

- If short-run profits are positive
  - Industry is attractive to potential entrants
  - Increase in supply leads to lower prices and profits

- If short-run profits are positive
  - Existing firms will shut down
  - Decrease in supply leads to higher prices and profits

With free entry and an unlimited number of potential entrants, profits approach zero in the long run

- Suppose that all firms have the same cost function $C(\cdot)$
- The zero-profit condition for each firm is

$$S_j(p)p - C\big(S_j(p)\big) = 0$$

- This condition can determine the long run equilibrium price p^L, regardless of the demand function
- With entry an exit, long-run supply becomes completely elastic
- Market clearing conditions determine the long-run number of firms

$$D\left(p^L\right) = nS_j\left(p^L\right)$$

- Suppose that all firms have the same cost function

$$C(q) = 8 + \frac{1}{2}q^2$$

- The corresponding individual short-run supply function is

$$S_j(p) = p$$

- The zero-profit condition is

$$p \cdot p - \left(8 + \frac{1}{2}p^2\right) = 0 \qquad \Rightarrow \qquad p^L = 4$$

- Suppose that the market demand is give by

$$D(p) = 20 - p$$

- The long-run market clearing condition is thus

$$20 - p^L = np^L$$

- Since we have determined that $p^L = 4$, it follows that

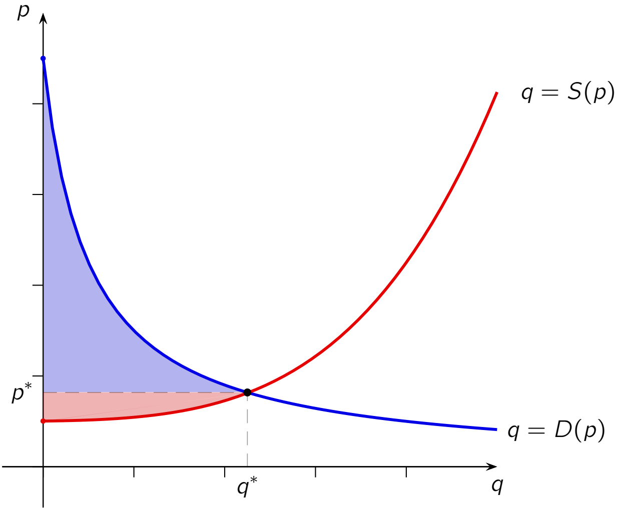
$$20 - 4 = n4 \qquad \Rightarrow \qquad n = 4$$

surplus

total surplus

- Total surplus is the sum of consumer and producer surplus
- Monetary measure of the welfare generated by the market
- It is maximized among all prices by the equilibrium price

maximum total surplus



equilibrium surplus = area above demand and below supply

- Suppose that the industry demand and supply are given by

$$D(q) = 8 - p \qquad \text{and} \qquad S(q) = p$$

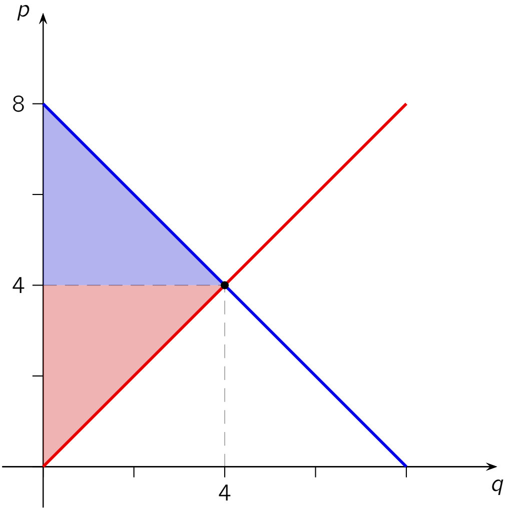
- Then the equilibrium price and quantity are given by

$$8 - p^* = p^* \qquad \Rightarrow \qquad p^* = 4 \text{ and } q^* = 4$$

- Consumer and producer surplus can be found using the figure (next slide)

$$\text{CS} = \text{PS} = \frac{4 \cdot 4}{2} = 8$$

maximum total surplus



equilibrium surplus with linear demand and supply

policy analysis

general equilibrium welfare

- A gasoline tax can have many different consequences
  - Change equilibrium price and quantity of gasoline
  - Decrease income of gasoline producers and their demand for other products
  - Decrease demand for gasoline compliments (e.g., muscle cars)
  - Increase demand for gasoline substitutes (e.g., renewables)
  - Decrease demand for gasoline inputs (e.g., crude oil, engineers)
  - Decrease cost of other industries that rely on similar inputs
- Partial equilibrium surplus fails to capture many of these effects

- Partial equilibrium — market for one good
- General equilibrium — markets for all goods and services in the economy
- This course — 2×2 pure-exchange economy
  - Two products (Milk and Salad)
  - Two consumers (Anna and Bob)
  - No production, only trade their initial endowments

- Suppose Anna and Bob's initial endowments are given in the table

	Anna	Bob	Total
Milk	4	1	5
Salad	1	9	10

- Cobb-Douglas preferences represented by

$$u_A = x_{AS}x_{AM} \qquad u_B = x_{BS}x_{BM}$$

- An allocation specifies how much each person consumes of each good

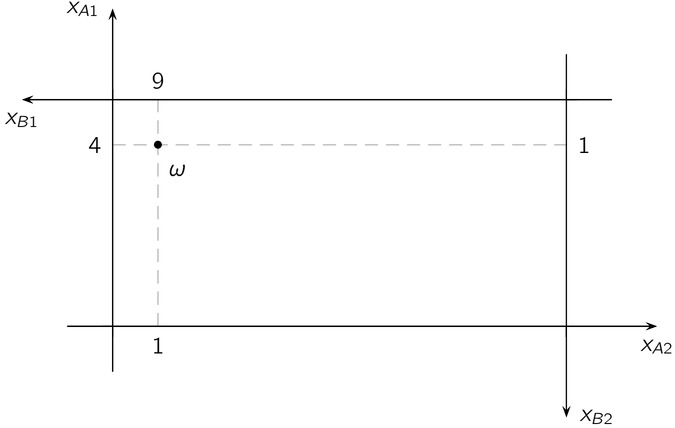
$$\mathbf{x} = (x_{AS}, x_{AM}, x_{BS}, x_{BM})$$

- All quantities must be non-negative
- Total consumption must equal total endowment

$$x_{AS} + x_{BS} = \omega_{AS} + \omega_{BS}$$
$$x_{AM} + x_{BM} = \omega_{AM} + \omega_{BM}$$

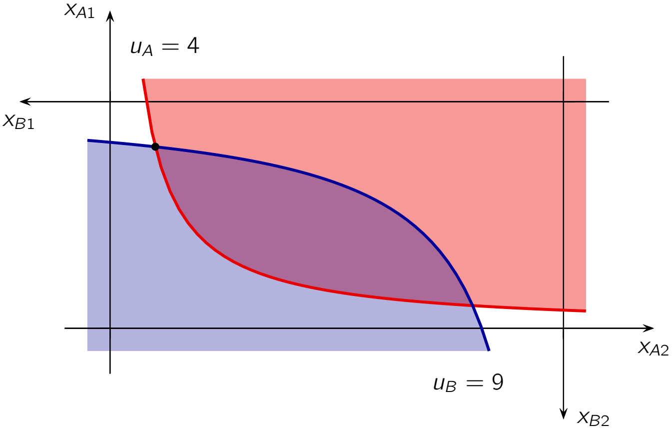
- Can write Bob's consumption in terms of Anna's consumption

Edgeworth box



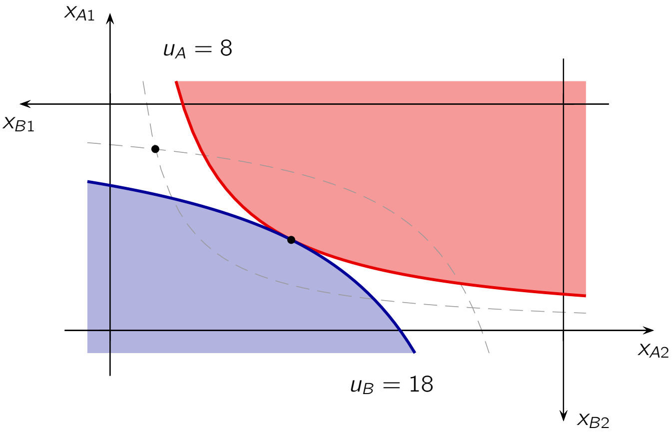
each point in the box corresponds to a feasible allocation

Pareto improvements



points between the indifference curves are Pareto improvements

Pareto efficiency



Pareto efficiency requires equitangency

Pareto efficiency

Under some conditions, an allocation the interior of the Edgeworth box is Pareto efficient if and only if

$$\frac{\text{MU}_{AM}}{\text{MU}_{AS}} = \frac{\text{MU}_{BM}}{\text{MU}_{BS}}$$

- Pareto efficient allocations must maximize Anna's utility subject to not making Bob any worse

$$\begin{aligned} \max_{x} \quad & u_A(x_{AS}, x_{AS}) \\ \text{st} \quad & u_B(\omega_S - x_{AS}, \omega_M - x_{AM}) \geq u_B^0 \end{aligned}$$

- First order condition is equitangency of indifference curves
- Boundary needs to be considered more carefully

Cobb-Douglas example

- In running example equitangency is

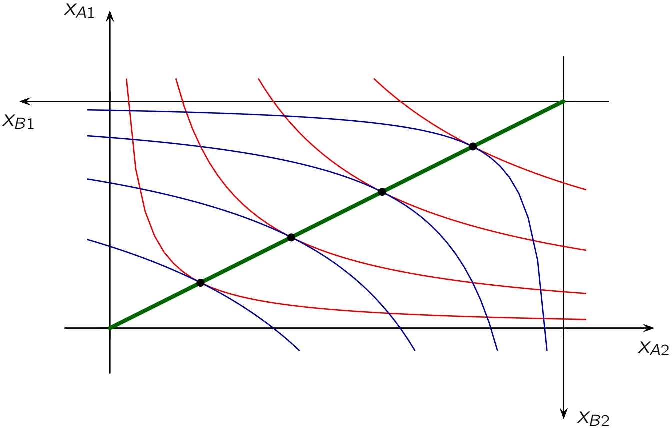
$$\frac{x_{AS}}{x_{AM}} = \frac{x_{BS}}{x_{BM}}$$

- Combining with feasibility yields

$$\frac{x_{AS}}{x_{AM}} = \frac{10 - x_{AS}}{5 - x_{AM}} \quad \Rightarrow \quad 5x_{AS} - x_{AS}x_{AM} = 10x_{AM} - x_{AS}x_{AM}$$

$$\Rightarrow \quad x_{AS} = 2x_{AM}$$

Cobb-Douglas example



Pareto frontier corresponds to the green line

competitive general equilibrium

- Consumers trade at centralized markets taking prices as given
- Prices adjust so that all markets clear

A **competitive general equilibrium** (CGE) for a pure exchange econonomy with n products consists of prices $(p_1^*, \ldots, p_n^*)$ and an allocation $\mathbf{x}$ such that

1. The consumption of each consumer i solves

$$\max_{x_{i1},\ldots,x_{in}} \quad u_i(\mathbf{x}_i) \qquad \text{st} \quad \sum_k p_k^* x_{ik} = \sum_k p_k^* \omega_{ik}$$

2. The market for each product k clears, that is

$$\sum_i x_{ik} = \sum_i \omega_{ik}$$

2×2 **pure exchange economy**

- In a 2×2 economy with well-behaved preferences (smooth, quasi-concave, and strictly increasing) the CGE are given by
  1. The first-order conditions of the consumer problems
  $$\frac{\mathrm{MU}_{AM}}{\mathrm{MU}_{AS}} = \frac{p_M}{p_S} = \frac{\mathrm{MU}_{BM}}{\mathrm{MU}_{BS}}$$
  2. The consumer budget constraints
  $$p_M x_{AM} + p_S x_{AS} = p_M \omega_{AM} + p_S \omega_{AS}$$
  $$p_M x_{BM} + p_S x_{BS} = p_M \omega_{BM} + p_S \omega_{BS}$$
  3. The market-clearing or feasibility conditions
  $$x_{AS} + x_{BS} = \omega_{AS} + \omega_{BS}$$
  $$x_{AM} + x_{BM} = \omega_{AM} + \omega_{BM}$$