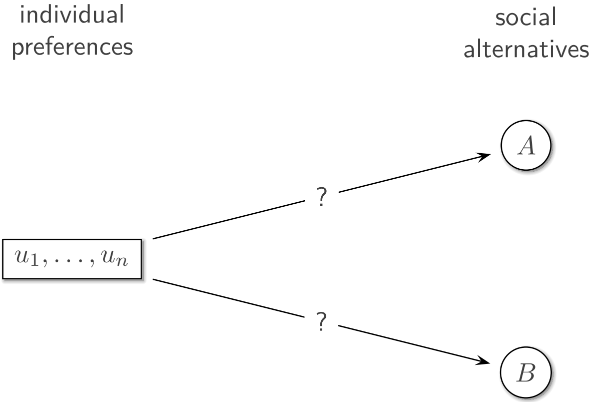
Institutions

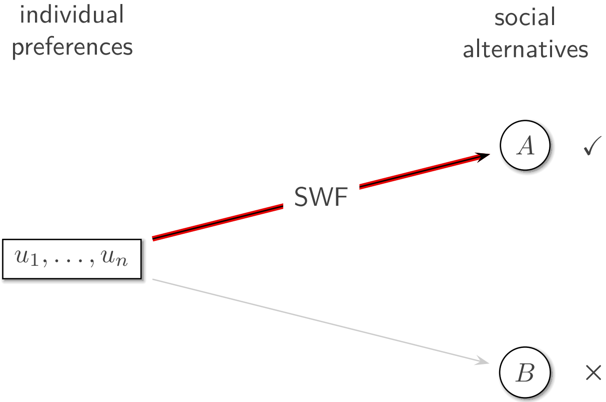
Bruno Salcedo

Economics 2261 · Intermediate Micro II · Winter 2020

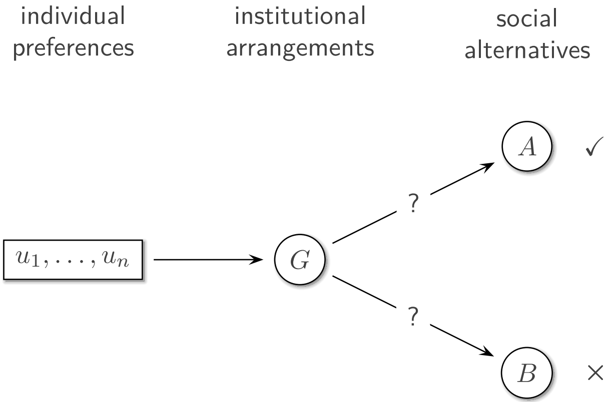
mechanism deisgn



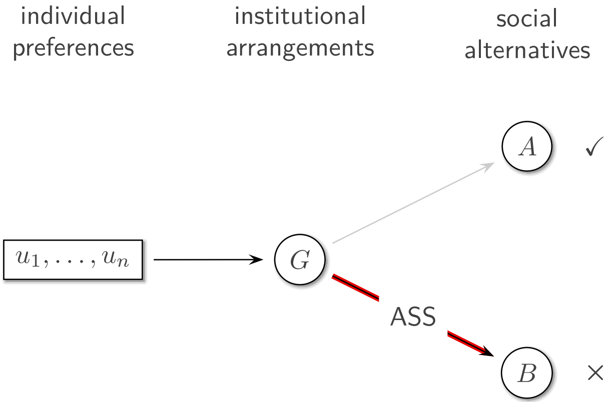
how should people behave?



how should people behave?



how do people behave given an institution?

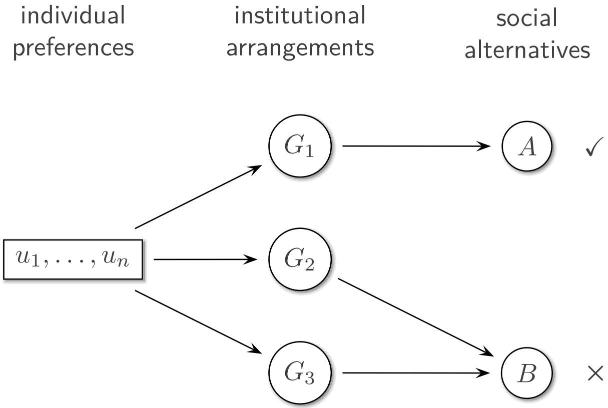


how do people behave given an institution?

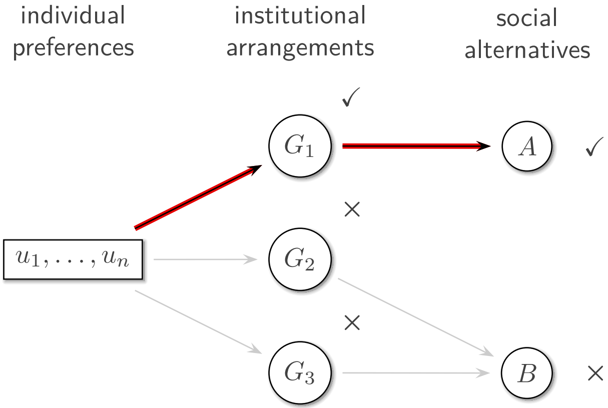
	C	D
C	$-1, -1$	$-6, 0$
D	$0, -6$	$-5, -5$

individually optimal $\not\Rightarrow$ socially optimal

mechanism design



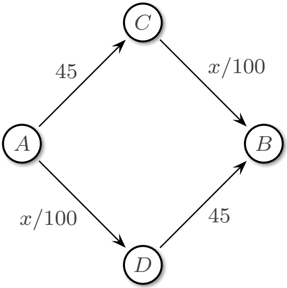
which institutions induce desired behavior?



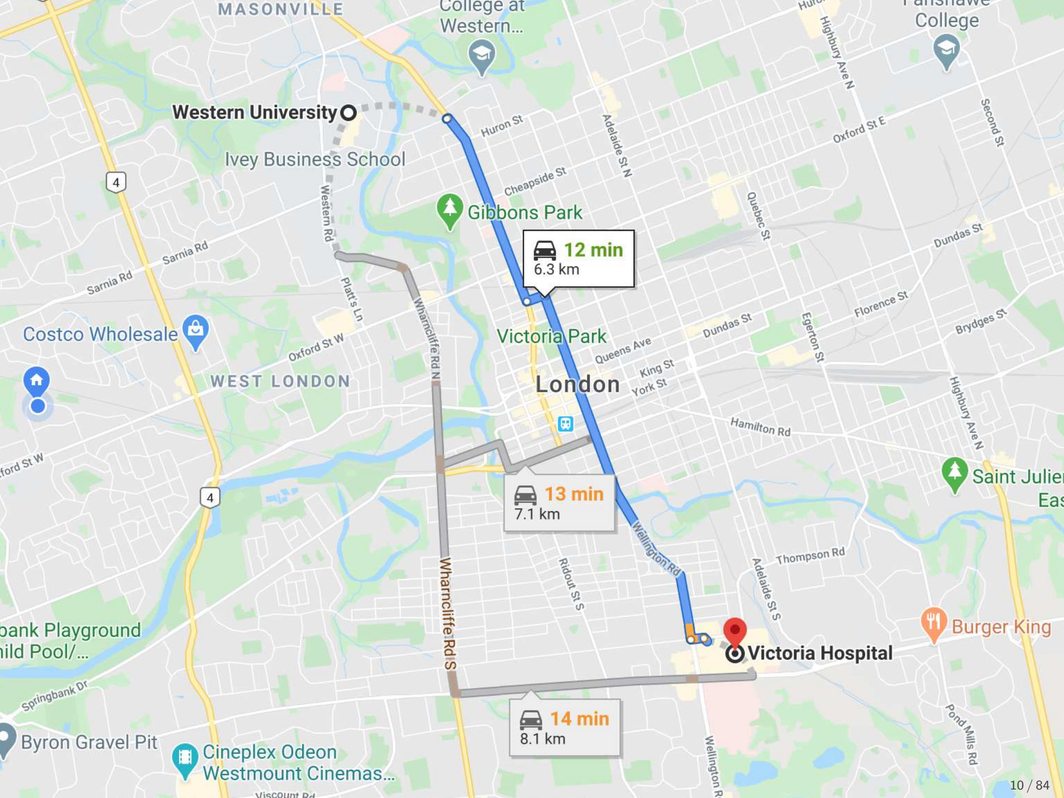
which institutions induce desired behavior?


50
Sarnia Rd

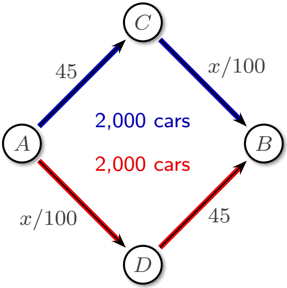
braess paradox



- 4,000 drivers need to go from A to B
- Segments AC and DB are wide but long
- Segments AD and CB are short but narrow

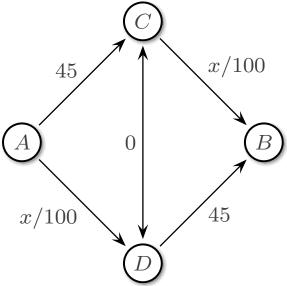

MASONVILLE
College at Western...
Fanshawe College
Western University
Ivey Business School
Huron St
Adelaide St N
Highbury Ave N
Second St
Oxford St E
Cheapside St
Gibbons Park
Quebec St
Dundas St
Western Rd
Sarnia Rd
Sarnia Rd
12 min
6.3 km
Platt's Ln
Wharncliffe Rd N
Victoria Park
Egerton St
Florence St
Brydges St
Costco Wholesale
Oxford St W
Queens Ave
Dundas St
WEST LONDON
London
King St
York St
Hamilton Rd
Highbury Ave N
Oxford St W
4
4
Saint Julien Eas
13 min
7.1 km
Wellington Rd
Thompson Rd
Adelaide St S
Ridout St S
Wharncliffe Rd S
Burger King
Victoria Hospital
bank Playground
hild Pool/...
Springbank Dr
14 min
8.1 km
Byron Gravel Pit
Cineplex Odeon
Westmount Cinemas...
Viscount Rd
Wellington Rd
Pond Mills Rd

traffic pattern



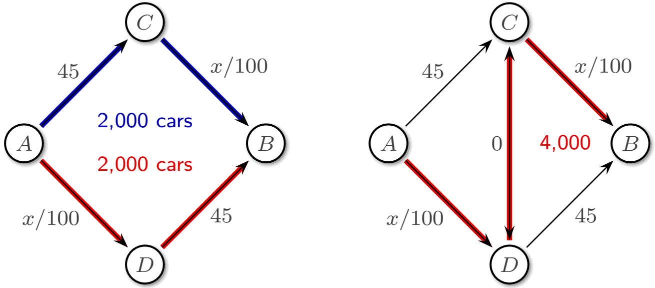
- Each driver chooses the fastest route taking traffic into account
- As a result, half the drivers take each route and takes 65 min

policy proposal



- Politician proposes a bridge connecting D to C
- How much should we pay for it?

Braess' paradox



- Now, all cars will take the route ADCB and take 80 min!

Adding resources to a network can worsen its performance

- Selfish (but normal) behavior—congestion externalities are not internalized
- New road concentrates drivers on the same route $\implies$ increases externalities

- A randomly added road has close to a 50-50 chance of worsening congestion
- Ring roads vs. though highways

- New roads can worsen traffic even without induced demand
- Closing/narrowing roads can improve traffic

- Political Economics issue—hard to implement non-intuitive policies


HanMir
My Bank

a roommates’ dilemma

CDN$ 999.99

prime

FREE Delivery by **Saturday**

Get it **Friday** if you order within 18 hours and 22 minutes and choose paid shipping at checkout.

In Stock.

Quantity: 1

Add to Cart

Buy Now

Ships from and sold by Amazon.ca.

Item arrives in packaging that reveals what's inside and can't be hidden. If this is a gift, consider shipping to a different address.

- Buy or not?
- How to split cost? $t_F + t_G = 1000$

- No resale value
- No maintenance
- No restricting usage
- No monitoring of usage

How would you and your roommate make this decision?

proposed mechanisms

- Buy only if both are willing to split cost 50-50
- Whoever drinks more coffee/wants it more pays proportionally more
- Frankie buys the machine and Gary compensates her depending on how much espresso he plans to drink
- Each roommate buys their own machine without sharing
- Alternated bargaining

Which is the best mechanism to use?

- Buy only if both are willing to split cost 50-50
- Whoever drinks more coffee/wants it more pays proportionally more
- Frankie buys the machine and Gary compensates her depending on how much espresso he plans to drink
- Each roommate buys their own machine without sharing
- Alternated bargaining

Can we find at least one Pareto efficient mechanism?

- Utility from buying = value from using − money paid

$$u_i = \begin{cases} v_i - t_i & \text{if buy} \\ 0 & \text{if not} \end{cases}$$

- t_i could be negative as long as $t_F + t_G = 1000$
- No-money burning (for now)

- quasilinear utility + monetary transfers implies

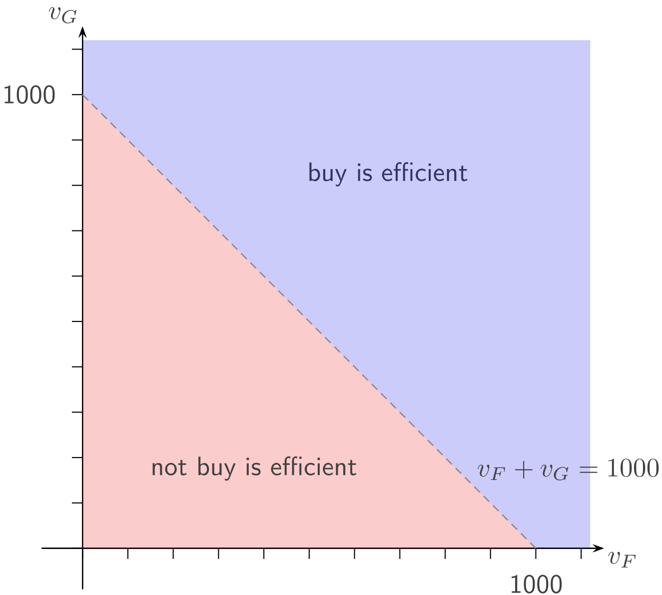
$$\text{Pareto} \iff \text{Utilitarian}$$

- Efficiency = maximizing sum of utilities

$$u_F + u_G = \begin{cases} v_F + v_G - 1000 & \text{if buy} \\ 0 & \text{if not} \end{cases}$$

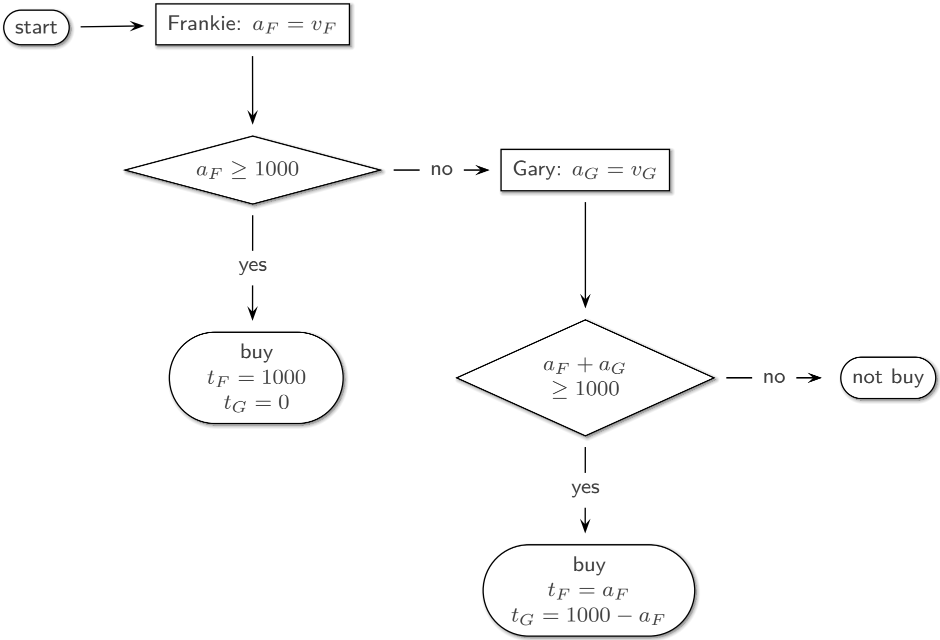
Efficiency — Buy if and only if $v_F + v_G \geq 1000$

efficiency



v_G
1000
buy is efficient
not buy is efficient
$v_F + v_G = 1000$
v_F
1000

an efficient mechanism

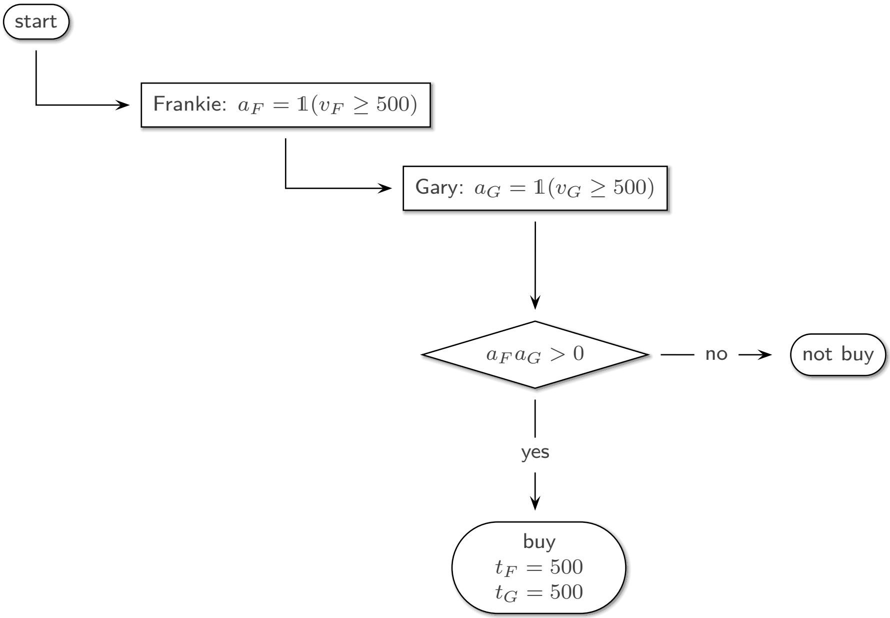


- Only Franky knows $v_F = 1{,}200$
- Only Gary knows $v_G = 750$
- The mechanism relies on truthful reporting $(a_i = v_i)$

- Suppose Franky knows $v_G \geq 300$
- If she reports truthfully she pays $t_F = 1{,}000$
- If she underreports $a_F = 700$ she only pays $t_F = 700$
- The machine would be bought either way

The proposed efficient mechanism is not incentive compatible

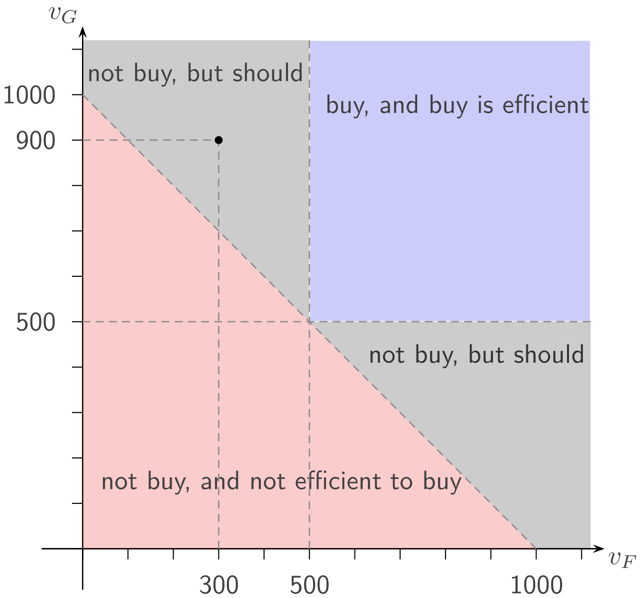
50-50 split


start
Frankie: $a_F = \mathbb{1}(v_F \geq 500)$
Gary: $a_G = \mathbb{1}(v_G \geq 500)$
$a_F a_G > 0$
no
not buy
yes
buy
$t_F = 500$
$t_G = 500$

50-50 split mechanism is incentive compatible

	1	0
1	$v_i - 500$	0
0	0	0

- $v_i > 500 \implies$ saying yes is weakly dominant
- $v_i < 500 \implies$ saying no is weakly dominant


v_G
1000
900
500
not buy, but should
buy, and buy is efficient
not buy, but should
not buy, and not efficient to buy
300
500
1000
v_F

- Efficient mechanism—not incentive compatible
- 50-50 split—incentive compatible but inefficient

Is there an efficient incentive-compatible mechanism?

the revelation principle

How to choose a public policy that affects different individuals with (typically) different preferences over policies, if the individual's preferences are **private information**?

- Set A of alternatives $a, b, \ldots$
- A set of individuals $i = 1, \ldots, n$
- For each individual i, a quasilinear utility function

$$u_i(a, t_i) = v_i(a) - t_i$$

- Pareto efficiency is equivalent to maximizing sum of values

$$\sum_i v_i(a)$$

Problem — It is often the case that the preferences of each individual are known only by the individual themself

- A mechanism consists of
  1. Set of actions or messages M_i for each i
  2. An allocation rule $\alpha(m_1, \ldots, m_n) \in \mathcal{A}$
  3. A transfer rule for each player $t_i(m_1, \ldots, m_n)$
- Mechanism + Preferences = Game
- Solve using cautiousness (for example)

- Optimal mechanism design—maximizing profits
- Efficient mechanism design—maximizing social welfare (Pareto)

> **Definition** — A mechanism is efficient if the predicted outcomes of the game always maximize $\sum_i v_i$

- Agents are asked to report their preferences
- Reports are made simultaneously and independently
- Alternative and transfers determined by $\alpha(\cdot)$ and $t(\cdot)$

> **Definition** — A direct mechanism is incentive-compatible if lying is weakly dominated by truth-telling.

revelation principle

Theorem — Restricting attention to incentive-compatible direct mechanisms is without loss of generality

the vickrey mechanism

- Anna inherited unwanted artwork
- Bob, Charlie, and David want it for personal use

	a	b	c	d
v_A	0	0	0	0
v_B	0	7	0	0
v_C	0	0	10	0
v_D	0	0	0	4

Vickrey mechanism

- Sealed-bid second-price auction (for a single object)
- Direct mechanism
  - Each buyer makes a bid m_i
  - Object is allocated to the buyer with the highest bid
  - The winner pays the second highest bid to the seller
  - Buyers only pay if they win

	a	b	c	d
v_A	0	0	0	0
v_B	0	7	0	0
v_C	0	0	10	0
v_D	0	0	0	4

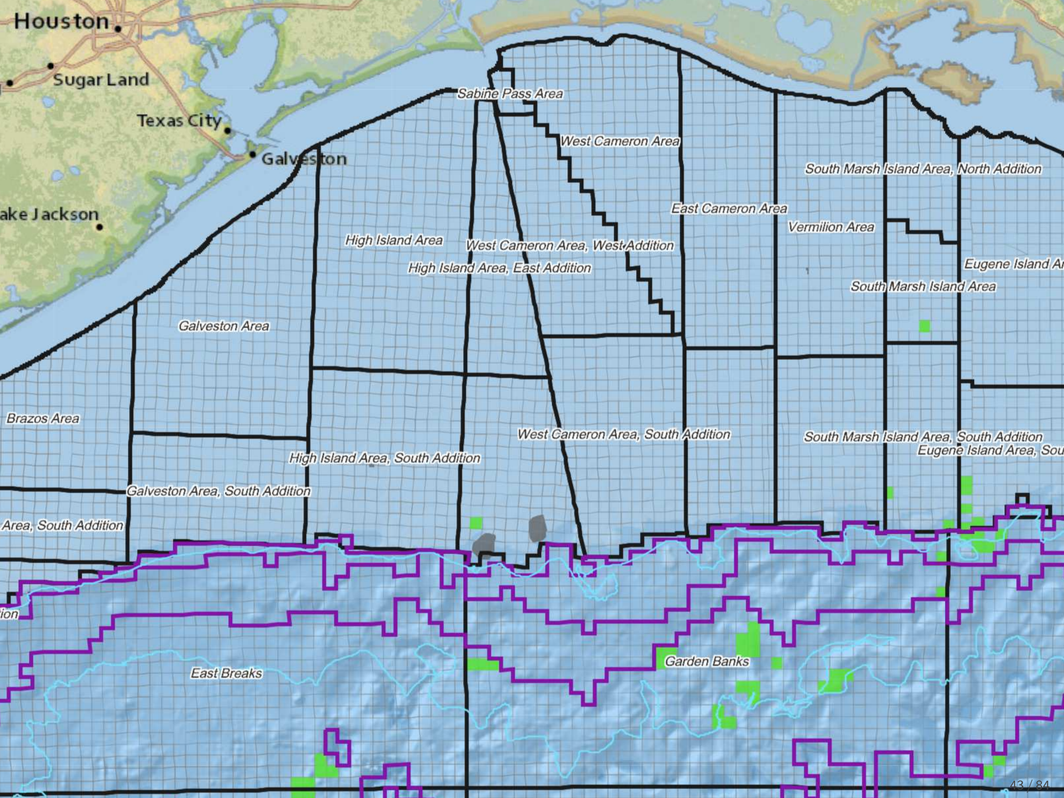
Charlie gets the artwork and pays $7 to Anna

Claim — Under some conditions, the Vickrey mechanism is efficient and incentive compatible

- Two important conditions: **private values** and **no externalities**

- Highest bid of j's opponents $p = \max\{m_j | j \neq i\}$
- Truth-telling weakly dominates overbidding and underbidding

	$m_i = v_i$	$m_i = \hat{v}_i > v_i$
$v_i < \hat{v}_i < p$	0	0
$p < v_i < \hat{v}_i$	$v_i - p$	$v_i - p$
$v_i < p < \hat{v}_i$	0	$v_i - p < 0$


Houston
Sugar Land
Texas City
Galveston
ake Jackson
Sabine Pass Area
West Cameron Area
South Marsh Island Area, North Addition
East Cameron Area
Vermilion Area
High Island Area
West Cameron Area, West Addition
High Island Area, East Addition
Eugene Island Ar
South Marsh Island Area
Galveston Area
Brazos Area
West Cameron Area, South Addition
South Marsh Island Area, South Addition
Eugene Island Area, Sou
High Island Area, South Addition
Galveston Area, South Addition
Area, South Addition
ion
Garden Banks
East Breaks

- The value of the oilfield v^* is the same for all bidders
- Bidders have noisy signals about the value
- Winner curse—winning reveals that others knew the value is low

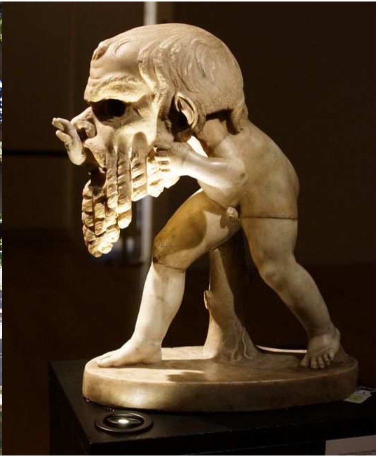
Claim — Bidders have incentives to underbid in a Vickrey auction with common values

- Field has oil $(v^* = 100)$ or not $(v^* = 0)$ with probability $1/2$ each
- Each bidder runs an independent test
  - With oil—test always comes back positive
  - Without oil—false positive with 1% probability

$$\Pr(\text{oil} \mid \text{positive test}) = \frac{0.5}{0.5 + 0.005} \approx 99\%$$

> If you bid a positive amount and someone (truthfully) bids zero, you realize that the field is worthless

externality

inefficiency from externalities

	a	b	c	d
v_A	0	0	0	0
v_B	0	7	0	0
v_C	0	0	10	0
v_D	0	0	-7	4

- Efficient outcome—Bob gets artwork
- Truth-telling—Charlie would get it
- Incentive compatibility—David has incentives to report $m_D = 11$

the vicrey–clarke–groves mechanism

- Vickery auction is efficient and incentive-compatible in some settings
- It fails with common values or consumption externalities
- It is not defined for roommate's problem
- For such cases we can use the Vickery–Clarke–Groves (VCG) mechanism

> Compensate/charge each member of society according to their contribution to the social welfare of others

bob's contribution to society

- Consider the efficient outcome in two situations
  - Bob is a member of society
  - Bob is not a member of society
- Compare the total utility of everyone except Bob
- The difference is called Bob's contribution to society

1. Maximize total welfare to find utilitarian alternative a^*
2. Compute total welfare from a^* of everyone except Bob

$$W_B^+ = \sum_{i \neq \mathrm{Bob}} v_i(a^*)$$

3. Find utilitarian alternative if Bob was not a member of society b^*
4. Compute total welfare from b^* of everyone except Bob

$$W_B^- = \sum_{i \neq \mathrm{Bob}} v_i(b^*)$$

5. Bob's contribution to society is the difference

$$W_B^+ - W_B^-$$

	b	c	d
v_B	7	0	0
v_C	0	10	0
v_D	0	0	4

- Single object with private vales and without externalities
- The efficient outcome is $a^* = c$
- Total welfare $\sum_i v_i(b) = 10$

bob's contribution to society

	b	c	d
v_B	7	0	0
v_C	0	10	0
v_D	0	0	4

- With Bob $W_B^+ = 10$
- Without Bob the best alternative is $b^* = c$
- Without Bob $W_B^- = 10$
- Bob's contribution to society is 0

charlie's contribution to society

	b	c	d
v_B	7	0	0
v_C	0	10	0
v_D	0	0	4

- With Charlie $W_C^+ = 0$
- Without Charlie the best alternative is $b^* = b$
- Without Charlie $W_C^- = 7$
- Charlie's contribution to society is -7

VCG mechanism

- Ask everyone to report their values
- Compute allocation and transfers using reported values $\hat{v}_i$
- Implement efficient allocation assuming truthful reporting

$$\alpha^{\mathrm{VCG}}(\hat{v}) = a^*(\hat{v})$$

- Individuals are compensated or charged by their social contribution

$$t_i^{\mathrm{VCG}}(\hat{v}) = W_i^+(\hat{v}) - W_i^-(\hat{v})$$

Claim — The Vickrey–CLarke–Groves mechanism is always efficient and incentive-compatible

	b	c	d
v_B	$v_B(b)$	0	0
v_C	0	$v_C(c)$	0
v_D	0	-7	$v_D(d)$

- For simplicity, assume that the size of the externality is known
- Bidders are only asked to report their private consumption value
- There are two interesting cases

- Suppose $v_C(c) - 7 > v_B(b) > v_D(d)$
- With Charlie—efficient to give the object to Charlie
- Without Charlie—efficient to give the object to Bob

$$t_C^{\text{VCG}} = \Big[v_B(c) + v_D(c)\Big] - \Big[v_B(b) + v_D(b)\Big] = -V_B(b) - 7$$

- VCG transfer = second-highest bid + externality

when Bob wins over Charlie

- Suppose $v_B(b) > v_C(c) - 7 > v_D(d)$
- With Bob—efficient to give the object to Bob
- Without Bob—efficient to give the object to Charlie

$$t_B^{\text{VCG}} = \Big[v_C(b) + V_D(b)\Big] - \Big[v_C(c) + V_D(d)\Big] = -V_C(c) + 7$$

- VCG transfer $=$ second-highest bid - externality

- Efficient by construction (under truthful reporting)
- Utility as a function of reports

$$u_i = v_i\big(a^*(\hat{v})\big) + t_i^{\mathrm{VCG}}(\hat{v})$$

- Substituting with VCG transfers

$$\begin{aligned} u_i &= v_i\left[a^*(\hat{v})\right] + W_i^+(\hat{v}) - W_i^-(\hat{v}) \\ &= \underbrace{v_i\left[a^*(\hat{v})\right] + \sum_{j\neq i} \hat{v}_j\left[a^*(\hat{v})\right]}_{\text{maximized if truthful}} - \underbrace{\sum_{i\neq j} \hat{v}_i\left[b^*(\hat{v})\right]}_{\text{independent of } \hat{v}_i} \end{aligned}$$

balancing the budget

- Budget balance—total transfers from the players must not generate a deficit

$$\sum_i t_i \geq 0$$

- Participation constraints—players have to be willing to participate

$$\mathbb{E}[\, u_i \,] \geq 0$$

VCG transfers in allocation problems

- VCG transfers in allocation problems

$$t_i^{\mathrm{VCG}}(\hat{v}) = \underbrace{-\sum_{j\neq i} \hat{v}_j\left(\alpha(\hat{v})\right)}_{\text{others' welfare}} + \underbrace{\sum_{j\neq i} \hat{v}_j\left(\alpha_{-i}(\hat{v}_{-i})\right)}_{\text{independent of } \hat{v}_i}$$

- Players have incentives to report truthfully and maximize welfare

$$u_i(\hat{v}_i) = \underbrace{v_i(\alpha(\hat{v})) + \sum_{j\neq i} v_j\left(\alpha(\hat{v})\right)}_{\text{total welfare}} - \underbrace{\sum_{j\neq i} v_j\left(\alpha^*(\hat{v}_{-i})\right)}_{\text{independent of } \hat{v}_i}$$

VCG transfers in general

- VCG transfers for general social choice problems

$$t_i^{\text{VCG}}(\hat{v}) = \underbrace{-\sum_{j \neq i} \hat{v}_j\left(\alpha(\hat{v})\right)}_{\text{others' welfare}} + \underbrace{H_i(\hat{v}_{-i})}_{\text{independent of } \hat{v}_i}$$

- Players have incentives to report truthfully and maximize welfare

$$u_i(\hat{v}_i) = \underbrace{v_i(\alpha(\hat{v})) + \sum_{j \neq i} v_j\left(\alpha(\hat{v})\right)}_{\text{total welfare}} - \underbrace{H_i(\hat{v}_{-i})}_{\text{independent of } \hat{v}_i}$$

- High $H(\hat{v}_{-i})$ helps with budget (or maximize revenue)

- Cannot be too high because of participation constraints

- Gary, Frankie, and Oscar the Owner
- Oscar's opportunity cost for selling $c_O = 1000$ is common knowledge

	buy	not
Gary	v_G	0
Frank	v_F	0
Oscar	-1000	0

efficient outcome

	buy	not
Gary	v_G	0
Frank	v_F	0
Oscar	-1000	0

Buy the machine if and only if $v_G + v_F > 1000$

- Suppose $v_F + v_G < 1000$
- The VCG transfers are

$$t_G^{\mathrm{VCG}} = H_G(v_F, v_O)$$

$$t_F^{\mathrm{VCG}} = H_F(v_G, v_O)$$

$$t_O^{\mathrm{VCG}} = H_O(v_G, v_F)$$

- Suppose $v_F + v_G < v_O$
- The roommate's participation constraints imply

$$H_G(v_F) \leq 0$$

$$H_F(v_G) \leq 0$$

$$H_O(v_G, v_F) \leq 0$$

when jointly buying is efficient

- Suppose $v_F < 1000$, $v_G < 1000$, and $v_F + v_G > 1000$
- The VCG transfers satisfy

$$t_G^{\text{VCG}} = 1000 - v_F + H_G(v_F)$$

$$t_F^{\text{VCG}} = 1000 - v_G + H_F(v_G)$$

$$t_O^{\text{VCG}} = -v_F - v_G + H_O(v_G, v_F)$$

- Suppose $v_F < 1000$, $v_G < 1000$, and $v_F + v_G > 1000$

- From the case when buying was inefficient we know

$$H_F(v_G) \leq 0 \quad \text{and} \quad H_G(v_F) \leq 0$$

- Therefore

$$t_G^{\text{VCG}} = 1000 - v_F + H_G(v_F) \leq 1000 - v_F$$

$$t_F^{\text{VCG}} = 1000 - v_G + H_F(v_G) \leq 1000 - v_G$$

$$t_G^{\text{VCG}} = -v_F - v_G + H_O(v_G, v_F)$$

- Suppose $v_F < 1000$, $v_G < 1000$, and $v_F + v_G > 1000$
- The VCG transfers satisfy

$$t_G^{\mathrm{VCG}} \leq 1000 - v_F$$

$$t_F^{\mathrm{VCG}} \leq 1000 - v_G$$

$$t_G^{\mathrm{VCG}} = -v_F - v_G + H_O(v_G, v_F)$$

- Suppose $v_F < 1000$, $v_G < 1000$, and $v_F + v_G > 1000$
- Oscar's participation constraint implies

$$-1000 + v_G + v_G - H_O(v_G, v_F) \geq 0$$

$$\implies H_O(v_G, v_F) \leq -1000 + v_G + v_G$$

- Therefore

$$t_G^{\text{VCG}} \leq 1000 - v_F$$

$$t_F^{\text{VCG}} \leq 1000 - v_G$$

$$t_G^{\text{VCG}} = -v_F - v_G + H_O(v_G, v_F) \leq -1000$$

- Suppose $v_F < 1000$, $v_G < 1000$, and $v_F + v_G > 1000$
- The VCG transfers satisfy

$$t_G^{\mathrm{VCG}} \leq 1000 - v_F$$

$$t_F^{\mathrm{VCG}} \leq 1000 - v_G$$

$$t_G^{\mathrm{VCG}} \leq -1000$$

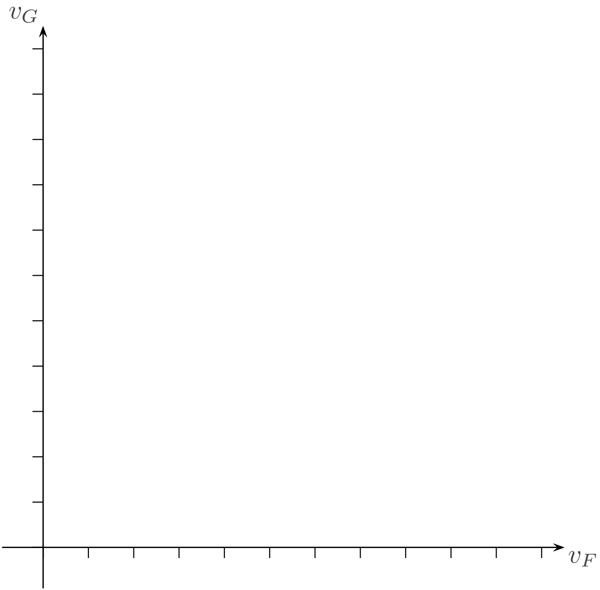
- And therefore the VCG mechanism runs a deficit

$$t_G^{\mathrm{VCG}} + t_F^{\mathrm{VCG}} + t_G^{\mathrm{VCG}} \leq 1000 - v_F - v_G < 0$$

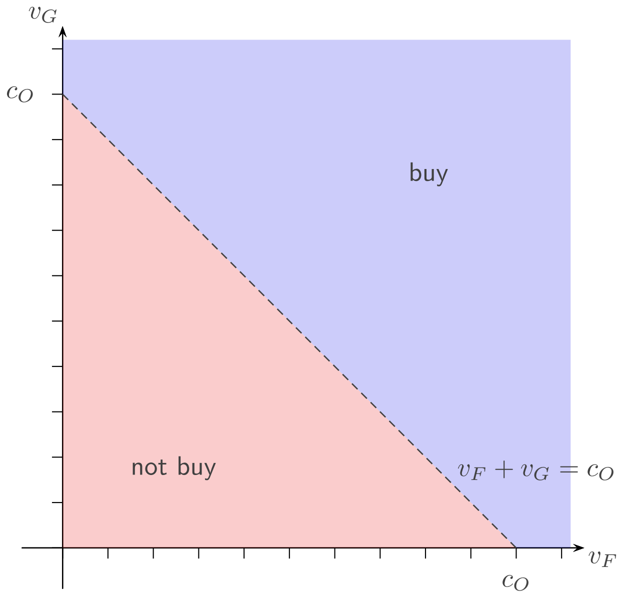
impossibility of first-best

	IC	PE	BB	IR
First mechanism	×	✓	✓	✓
50–50 split	✓	×	✓	✓
VCG	✓	✓	×	✓
VCG + forced tax	✓	✓	✓	×

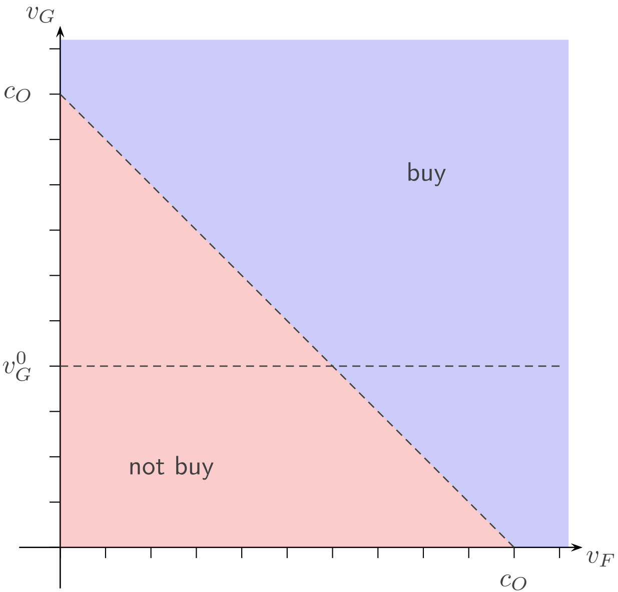
Can we find a mechanism satisfying all these conditions?



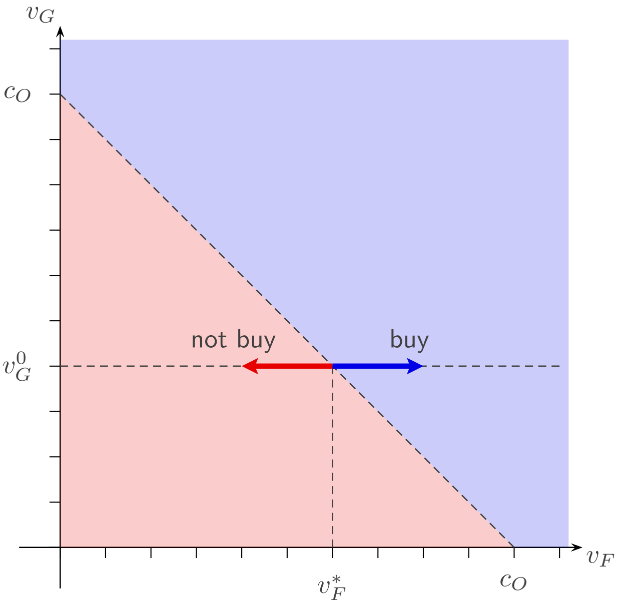
Pareto Efficiency completely determines the allocation rule



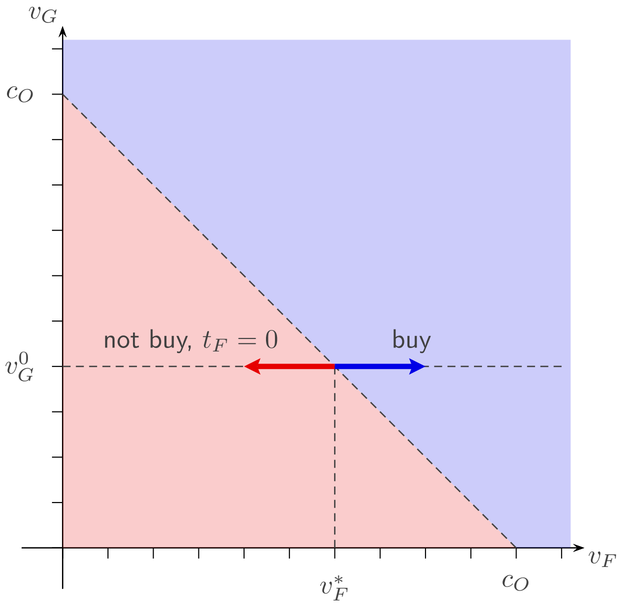
Fix some value v_G^0 for Gary and focus on Frank's inentives



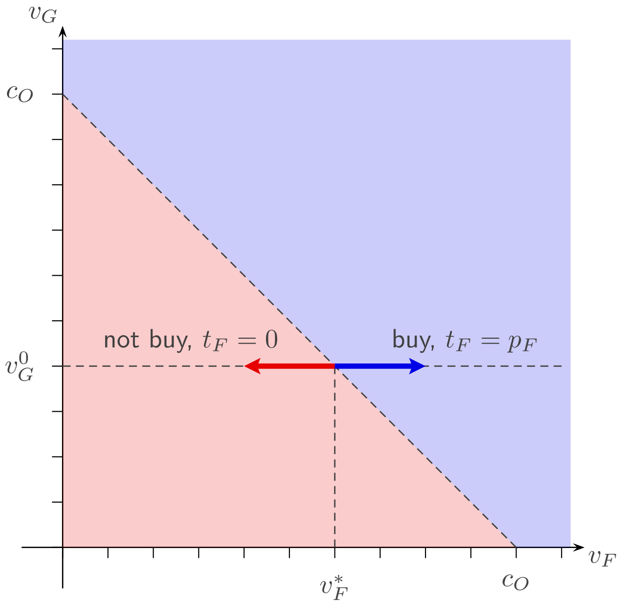
Efficient to buy if v_F is greater than $v_F^* := c_0 - v_G^0$



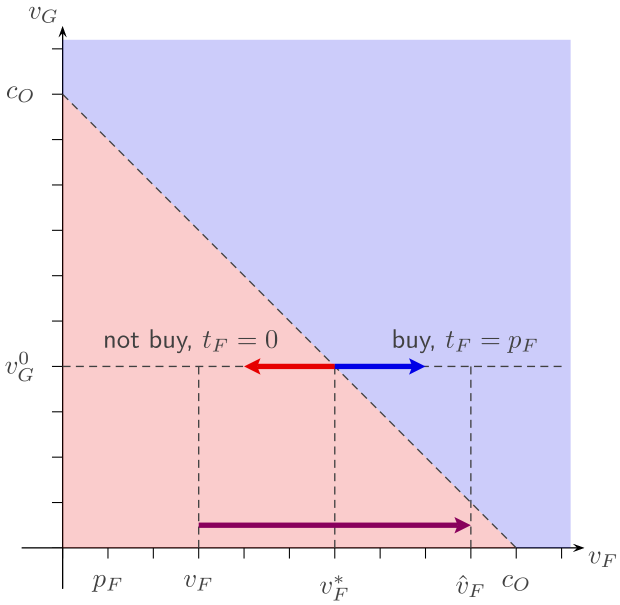
Frank's payment if they do not buy must be zero



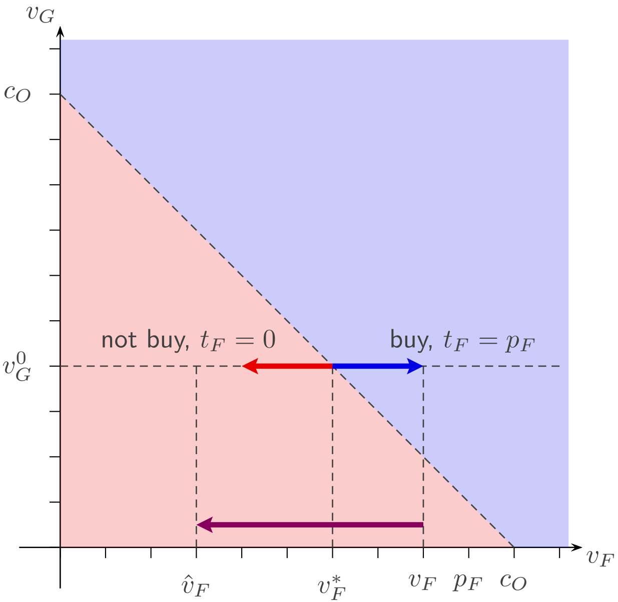
Frank's payment if they buy cannot deppend on his report
It must be a fixed price $p_F = p_F(v_G)$



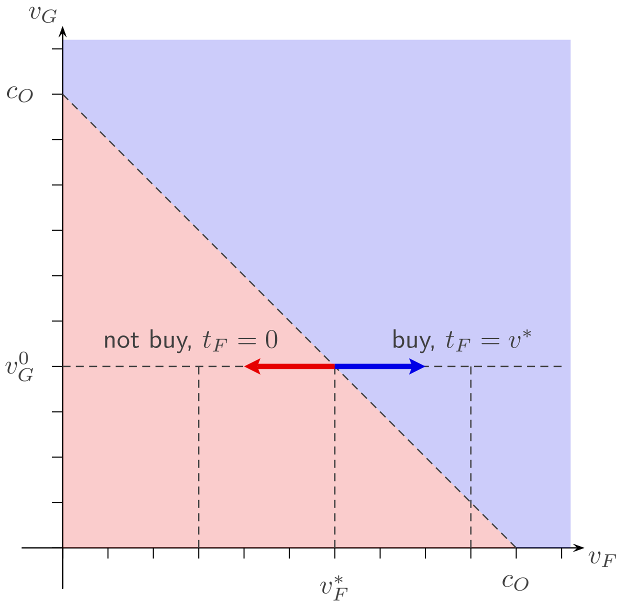
If $p_F < v^*$ and $p_F < v_F < v^*$, Frank wants to over-report



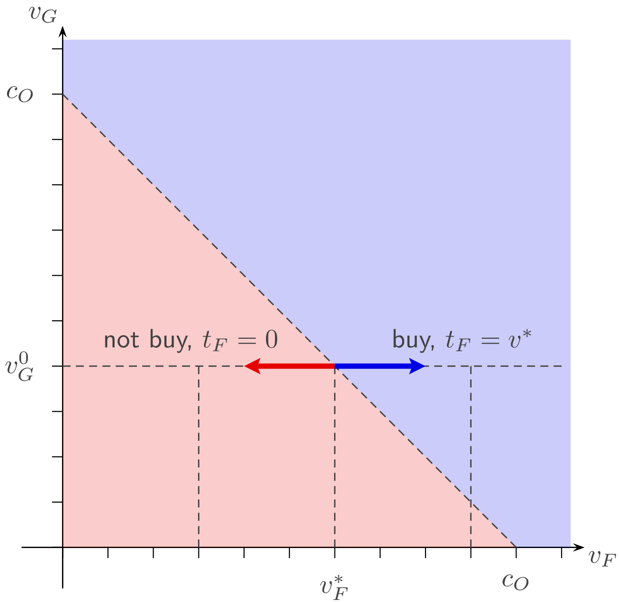
If $p_F < v^*$ and $p_F < v_F < v^*$, Frank wants to over-report



If $p_F > v^*$ and $v^* < v_F < p_F$, Frank wants to under-report



Only incentive compatible price is $p_F = v_F^* = c_O - v_G$



This is the VCG mechanism!

Claim — When the VCG mechanism runs a deficit, there are no mechanism satisfying PE, IC, BB, and IR.

Claim — There is no efficient mechanism for the provision of public goods that never runs a deficit and satisfies participation constraints.

next time we will discuss what to do when the first-best is impossible