ECON306 – Quiz 1

2014 · 5 · 17

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There are 4 questions. You have 40min to answer all of them. Don't forget to write your name and PSU ID (e.g. bxs5142) on all the pages that you want to be graded.

1. [30 pts] Consider the random variables x, y with the following joint distribution

	$y=-1$	$y=0$	$y=1$
$x=1$	1/8	3/8	1/8
$x=0$	1/8	1/8	1/8

(a) Find the expectation of x

$$\begin{aligned}\Pr(x=1) &= \Pr(x=1\ \&\ y=-1)+\Pr(x=1\ \&\ y=0)+\Pr(x=1\ \&\ y=1)\\ &= \frac{1}{8}+\frac{3}{8}+\frac{1}{8}=\frac{5}{8}\end{aligned}$$

$$\mathbb{E}[x] = 1\cdot\Pr(x=1)+0\cdot\Pr(x=0)=\Pr(x=1)=\frac{5}{8}$$

(b) Find the expectation of y

$$\Pr(y=1)=\Pr(y=1\ \&x=1)+\Pr(y=1\ \&x=0)=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

$$\Pr(y=-1)=\Pr(y=-1\ \&x=1)+\Pr(y=-1\ \&x=0)=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

$$\begin{aligned}\mathbb{E}[y] &= -1\cdot\Pr(y=-1)+0\cdot\Pr(y=0)+1\cdot\Pr(y=1)\\ &= \Pr(y=1)-\Pr(y=-1)=\frac{1}{4}-\frac{1}{4}=0\end{aligned}$$

(c) Find the expectation of the product xy

The product xy can take three values 1 (if $x = 1$ and $y = 1$), -1 (if $x = 1$ and $y = -1$) and 0 (if $x = 0$ or $y = 0$).

$$\Pr(xy = 1) = \Pr(x = 1 \& y = 1) = \frac{1}{8}$$
$$\Pr(xy = -1) = \Pr(x = 1 \& y = -1) = \frac{1}{8}$$

$$\mathbb{E}[xy] = -1 \cdot \Pr(xy = -1) + 0 \cdot \Pr(xy = 0) + 1 \cdot \Pr(xy = 1)$$
$$= \Pr(y = 1) - \Pr(y = -1) = \frac{1}{8} - \frac{1}{8} = 0$$

(d) Find the correlation between x and y

First we find the covariance of x and y

$$\sigma_{x,y} = \mathbb{E}[xy] - \mathbb{E}[x]\,\mathbb{E}[y] = 0 - \frac{5}{8} \cdot 0 = 0$$

The correlation between x and y is thus

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{0}{\sigma_x \sigma_y} = 0$$

(e) Are x and y independent? Justify your answer

No. For instance, notice that:

$$\Pr(x = 1 | y = 1) = \frac{1}{2} \neq \frac{5}{8} = \Pr(x = 1)$$

(f) Find the probability of the event that the product xy is different from 0

From the answer to (c) we have

$$\Pr(xy \neq 0) = \Pr(xy = 1) + \Pr(xy = -1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

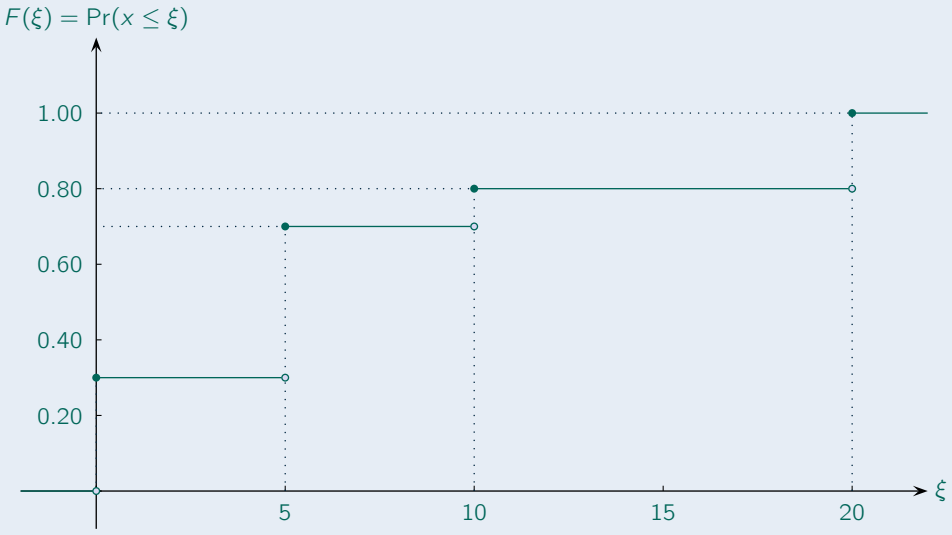
2. [30pts] Consider the random variables x, y with the following joint distribution

	$y=-1$	$y=\sqrt{2}$	$y=\pi$
$x=0$	0.2	0.01	0.09
$x=5$	0.1	0.1	0.2
$x=10$	0.025	0.075	0
$x=20$	0.05	0.05	0.1

(a) Find the marginal distribution of x

$$\begin{aligned}\Pr(x=0) &= \Pr(x=1 \,\&\, y=-1) + \Pr(x=1 \,\&\, y=\sqrt{2}) + \Pr(x=1 \,\&\, y=\pi)\\ &= 0.2 + 0.01 + 0.09 = 0.3\\ \Pr(x=5) &= \Pr(x=5 \,\&\, y=-1) + \Pr(x=5 \,\&\, y=\sqrt{2}) + \Pr(x=5 \,\&\, y=\pi)\\ &= 0.1 + 0.1 + 0.2 = 0.4\\ \Pr(x=10) &= \Pr(x=10 \,\&\, y=-1) + \Pr(x=10 \,\&\, y=\sqrt{2}) + \Pr(x=10 \,\&\, y=\pi)\\ &= 0.025 + 0.075 + 0 = 0.1\\ \Pr(x=20) &= \Pr(x=20 \,\&\, y=-1) + \Pr(x=20 \,\&\, y=\sqrt{2}) + \Pr(x=20 \,\&\, y=\pi)\\ &= 0.05 + 0.05 + 0.1 = 0.2\end{aligned}$$

(b) Find the marginal *cumulative* distribution of x



(c) Find the expectation of x

$$\begin{aligned}\mathbb{E}[x] &= 0 \cdot \Pr(x=0) + 5 \cdot \Pr(x=5) + 10 \cdot \Pr(x=10) + 20 \cdot \Pr(x=20)\\ &= 0 \cdot \frac{3}{10} + 5 \cdot \frac{4}{10} + 10 \cdot \frac{1}{10} + 20 \cdot \frac{2}{10}\\ &= 0 + 2 + 1 + 4 = 7\end{aligned}$$

(d) Find the expectation of x^2

$$\begin{aligned}\mathbb{E}\left[x^2\right] &= 0^2 \cdot \Pr(x=0) + 5^2 \cdot \Pr(x=5) + 10^2 \cdot \Pr(x=10) + 20^2 \cdot \Pr(x=20) \\ &= 0 \cdot \frac{3}{10} + 25 \cdot \frac{4}{10} + 100 \cdot \frac{1}{10} + 400 \cdot \frac{2}{10} \\ &= 0 + 10 + 10 + 80 = 100\end{aligned}$$

(e) Find the variance of x

$$\mathbb{V}[x] = \mathbb{E}\left[x^2\right] - (\mathbb{E}[x])^2 = 100 - 7^2 = 51$$

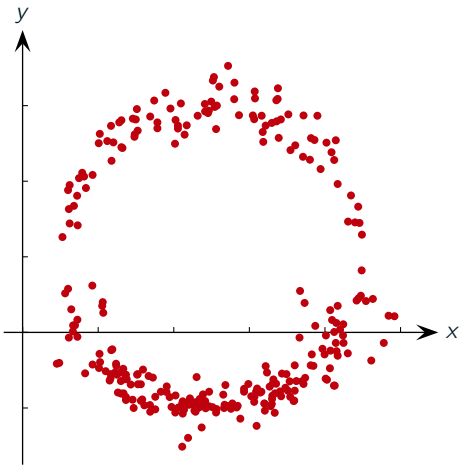
(f) Find the conditional distribution of y conditional on $x = 5$

$$\begin{aligned}\Pr(y=-1|x=5) &= \frac{\Pr(y=-1 \text{ and } x=5)}{\Pr(x=5)} = \frac{0.1}{0.4} = \frac{1}{4} \\ \Pr(y=\sqrt{2}|x=5) &= \frac{\Pr(y=\sqrt{2} \text{ and } x=5)}{\Pr(x=5)} = \frac{0.1}{0.4} = \frac{1}{4} \\ \Pr(y=\pi|x=5) &= \frac{\Pr(y=\pi \text{ and } x=5)}{\Pr(x=5)} = \frac{0.2}{0.4} = \frac{1}{2}\end{aligned}$$

3. [20pts] For each of the following four scatterplots involving random samples for variables x and y

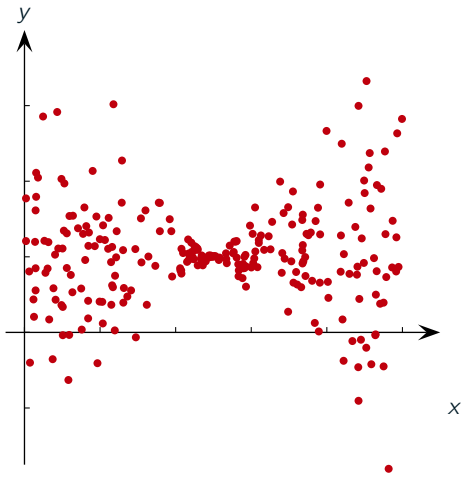
(a) Does the correlation of x and y appear to be positive, negative or close to 0?

(b) Do x and y appear to be independent?



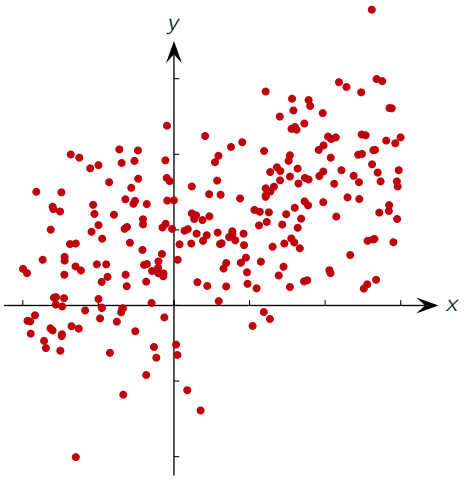
Correlation: Close to 0

Independence: NOT independent



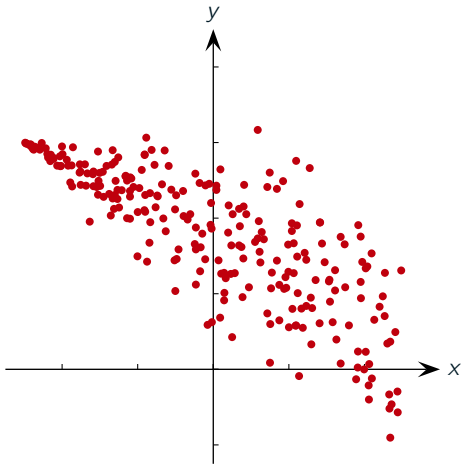
Correlation: Close to 0

Independence: NOT independent



Correlation: Positive

Independence: NOT independent



Correlation: Negative

Independence: NOT independent

4. [20pts] Which of the following objects are random (R) and which are deterministic (D)?

Random sample	(R)	D
Statistic	(R)	D
Empirical distribution	(R)	D
Estimate	R	(D)
Sample mean	(R)	D
Bias of an estimator	R	(D)
Estimator	(R)	D
Cumulative probability distribution	R	(D)
Sampling distribution	R	(D)
Expectation	R	(D)