Modeling Strategic Environments 1

Extensive form games

Watson §2, pages 11-23

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Extensive form games

- In order to fully describe a strategic environment we must specify:
  1. The players involved
  2. The actions that each player can take at each point of the game
  3. The information that each player has at the moment of making his choices
  4. The payoffs resulting from each combination of choices
- An extensive form game is a mathematical object that describes all these items.

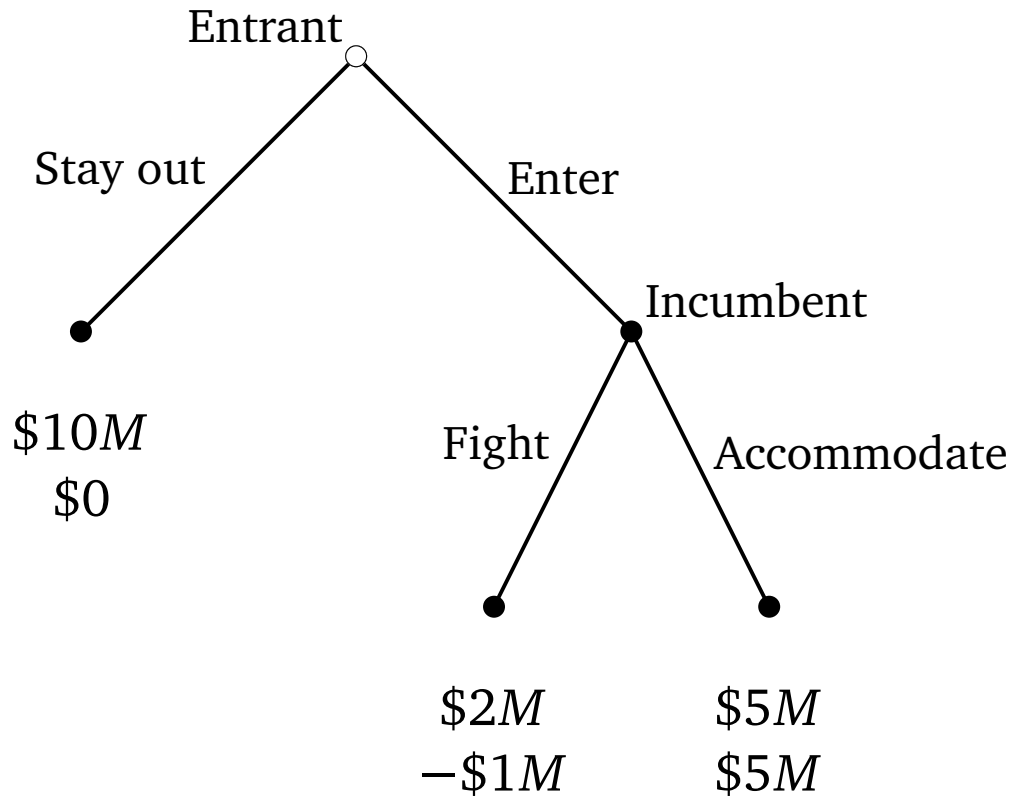
Example: entrance deterrence

Description

- Consider a market that is originally serviced by a monopolist firm which we call the *incumbent*
- A new potential *entrant* is considering to enter the market
- If the entrant stays out of the market, the incumbent can exploit its monopoly power to obtain high profits, say $\$10M$
- If the entrant enters the market then the incumbent must choose between two options:
  - It can choose to *fight* the entrant by adopting aggressive policies
  - It can choose to *accommodate* the entrant and share the profits
- If the incumbent decides to accommodate, it shares the monopoly profits with the entrant and they each receive a payoff of $\$5M$
- If the incumbent decides to fight, the entrant will suffer losses of $-\$1M$ but the incumbents profits will be reduced to $\$2M$

Example: entrance deterrence

Extensive from game

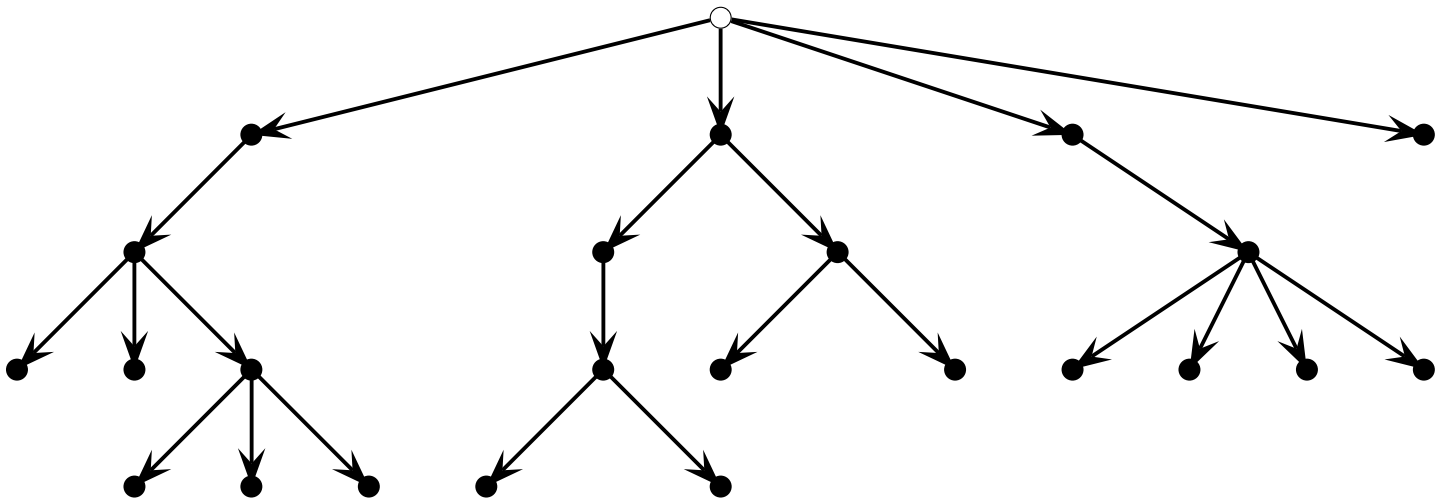


Trees

Definition

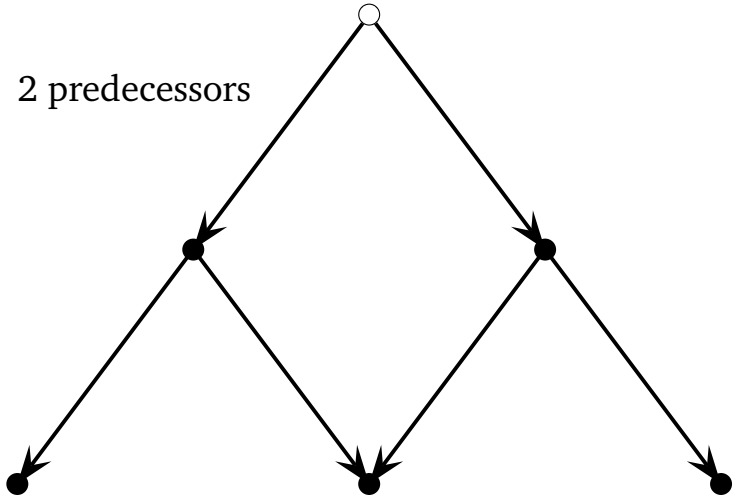
A tree is a mathematical object consisting of nodes connected by *directed* branches satisfying three properties:

- There is a unique node, called root, with no incoming branches
- Every node other than the root has a *unique* incoming branch
- There is a unique path connecting *any* two nodes

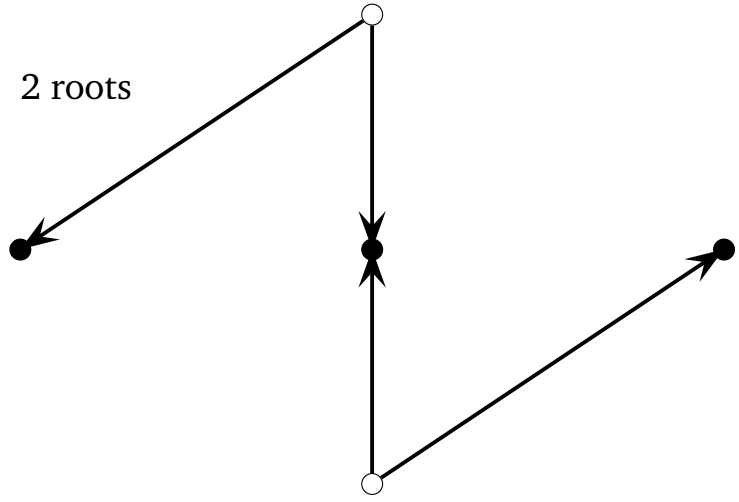


Example: things that are **NOT** trees

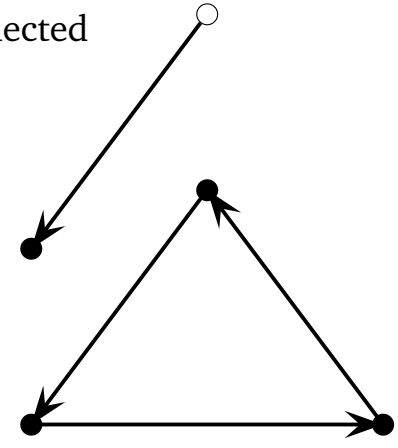
2 predecessors



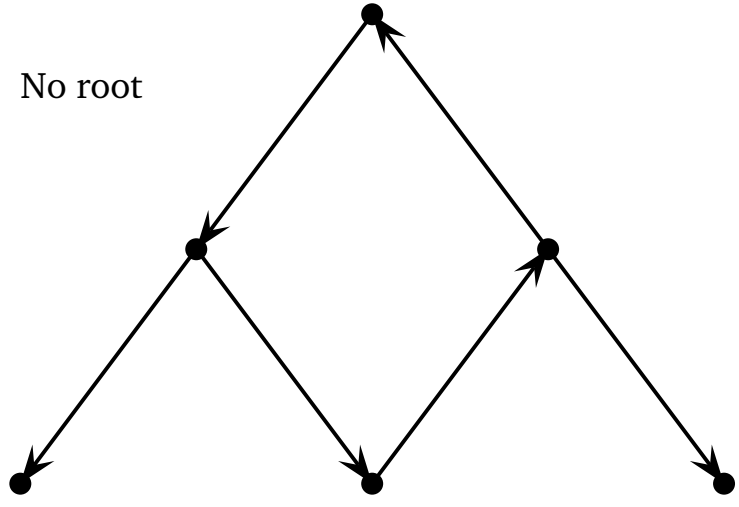
2 roots



Not connected

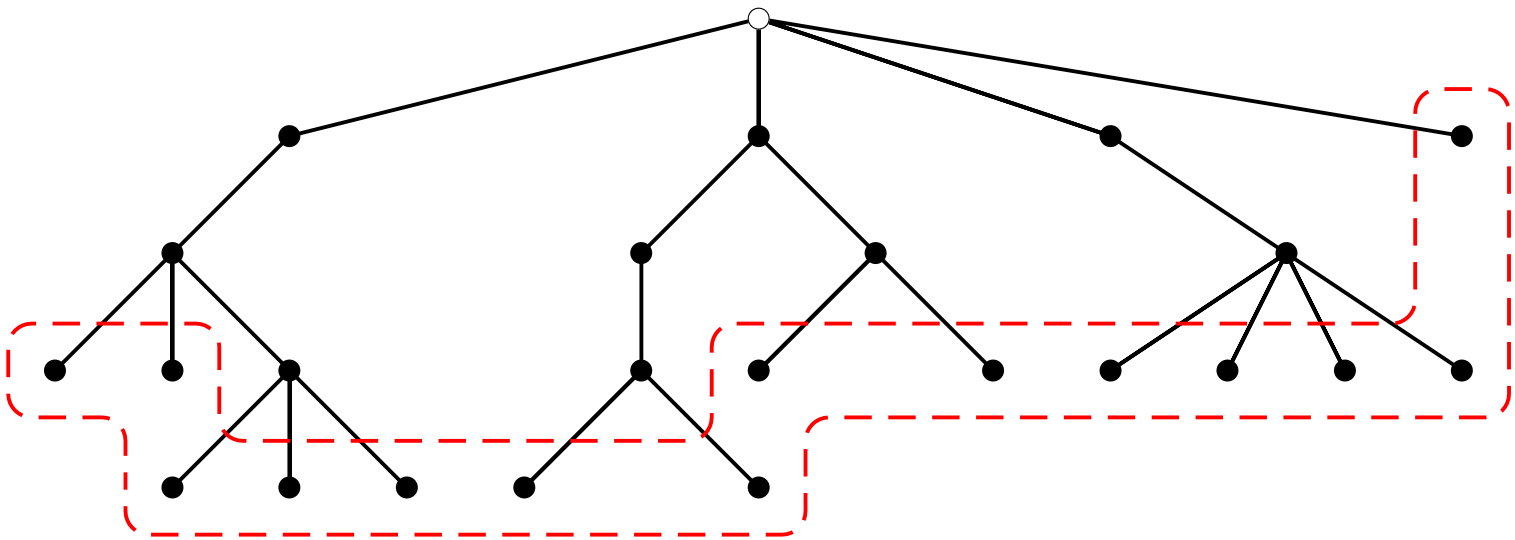


No root



Terminal and decision nodes

- We use trees to model the dynamic structure of the game.
- Each node represents a moment in the game:
  - A terminal node is a node without outgoing branches, it represents a final stage of the environment with no more choices to be made
  - Every other node is a decision node representing an intermediate stage at which some player gets make a choice



Extensive form games with perfect information

Definition

An extensive form game *with perfect information* is a mathematical object consisting of:

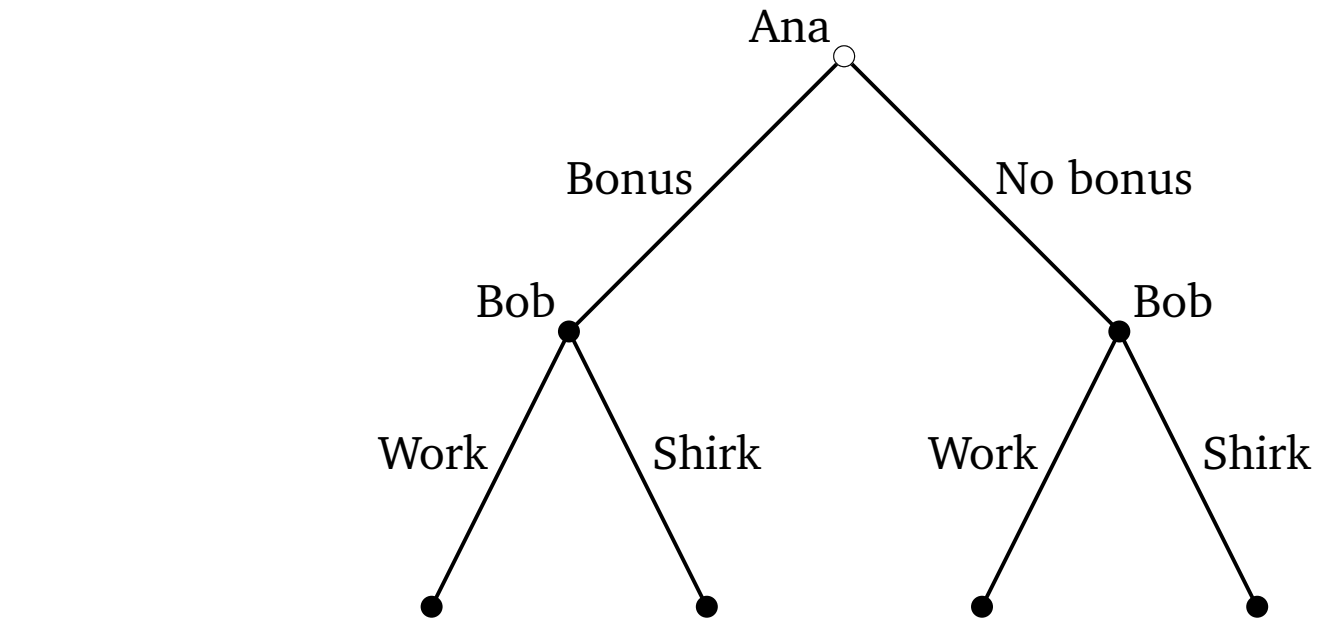
1. A set of players involved in the environment
2. A game tree representing the dynamic structure of choices
3. A specification of the player that gets to move at each decision node
4. A function that assigns a payoff for each player at each terminal node

Example: performance bonuses

- Suppose that Ana owns a company and Bob is one of her employees
- During a given year Bob can choose whether to work very hard or to shirk
- Working hard requires costly effort from Bob worth $100
- At the end of the year Bob receives a salary of $100 independently of whether he worked or shirked
- Ana's payoff depends on how well the company does:
  - If Bob works, the company does well and Ana gets $500
  - If Bob shirks, the company does poorly and Ana only gets $200
- At the beginning of the year Ana has the option of promising promise Bob a bonus of $150, conditional on him working hard

Example: performance bonuses

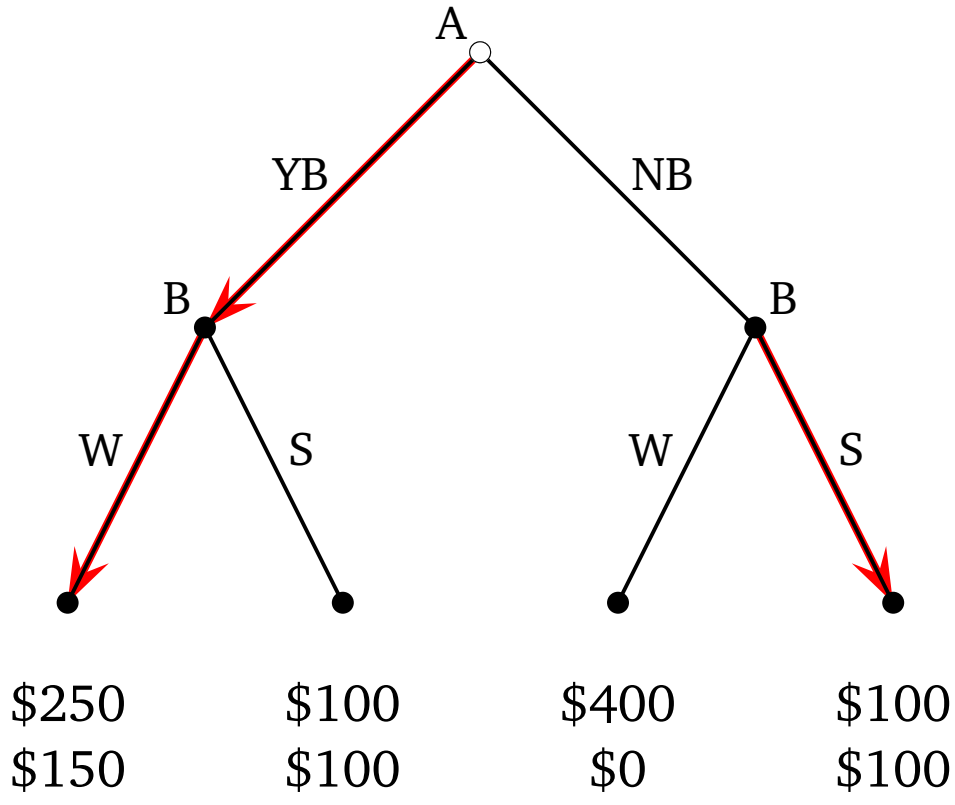
Extensive form game



+Profits	+$500	+$200	+$500	+$200
–Wage	–$100	–$100	–$100	–$100
–Bonus	–$150	–	–	–
Ana's payoff	$ 250	$100	$400	$100
+Wage	+$100	+$100	+$100	+$100
–Effort	–$100	–	–$100	–
+Bonus	+$150	–	–	–
Bob's payoff	$150	$100	$0	$100

Example: performance bonuses

Sneak preview: backward induction



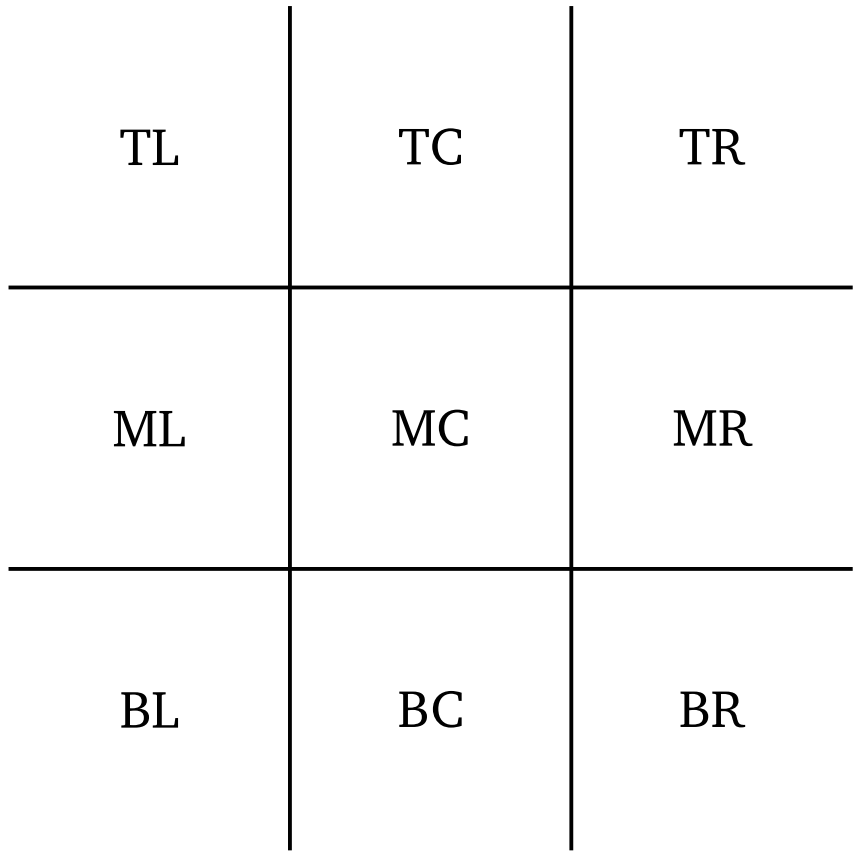
Example: Tic-Tac-Toe

Game rules

- Tic-tac-toe is a board game played on a 3 by 3 grid.
- There are two players, say Ana and Bob, that alternate taking turns
- On each of her turns, Ana picks a free space and marks it with a circle.
- On each of his turns, Bob picks a free space and marks it with a cross.
- A player wins the game if he/she gets three marks on the same horizontal, vertical or diagonal line.
- The winner's payoff is $+1$, the loser's payoff is -1 and both players get 0 if the game ends with a draw.

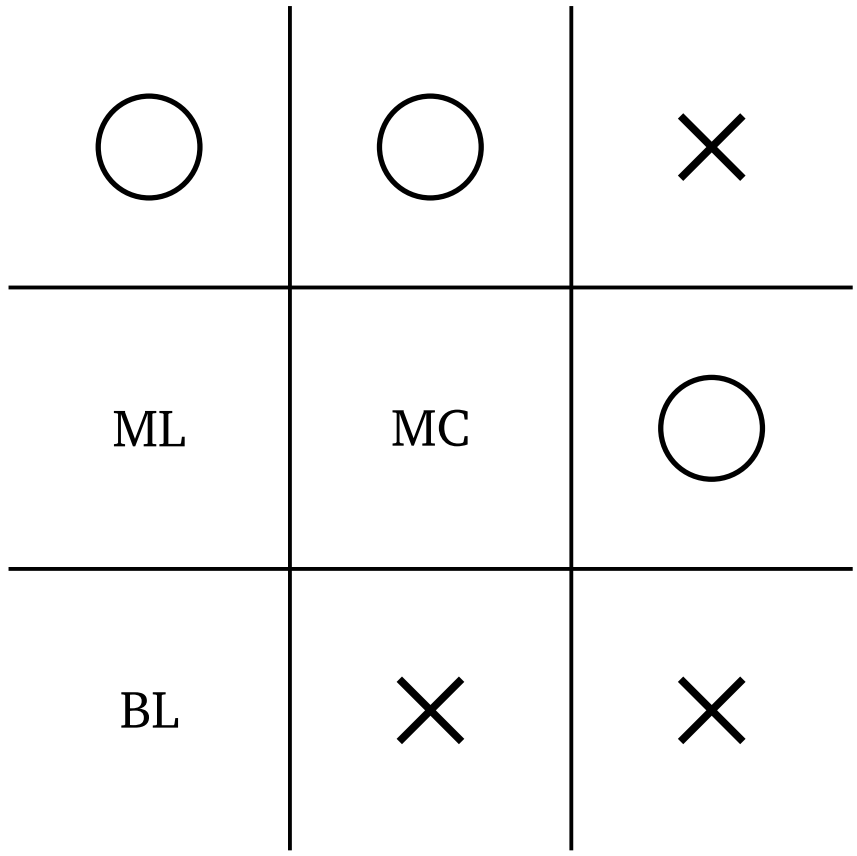
Example: Tic-Tac-Toe

The board



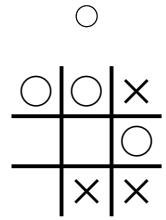
Example: Tic-Tac-Toe

A sample play



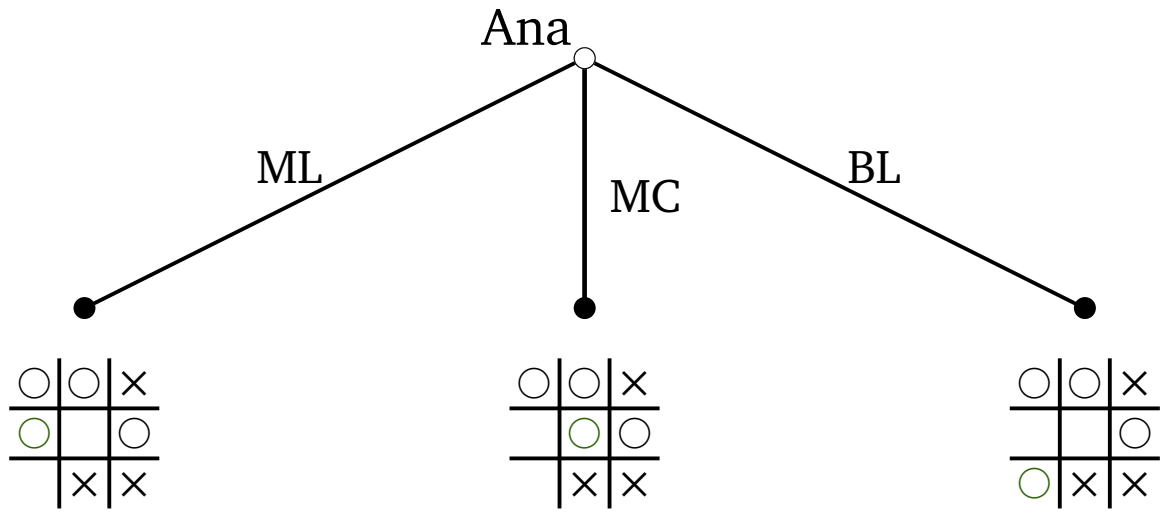
Example: Tic-Tac-Toe

Modeling the end of the game



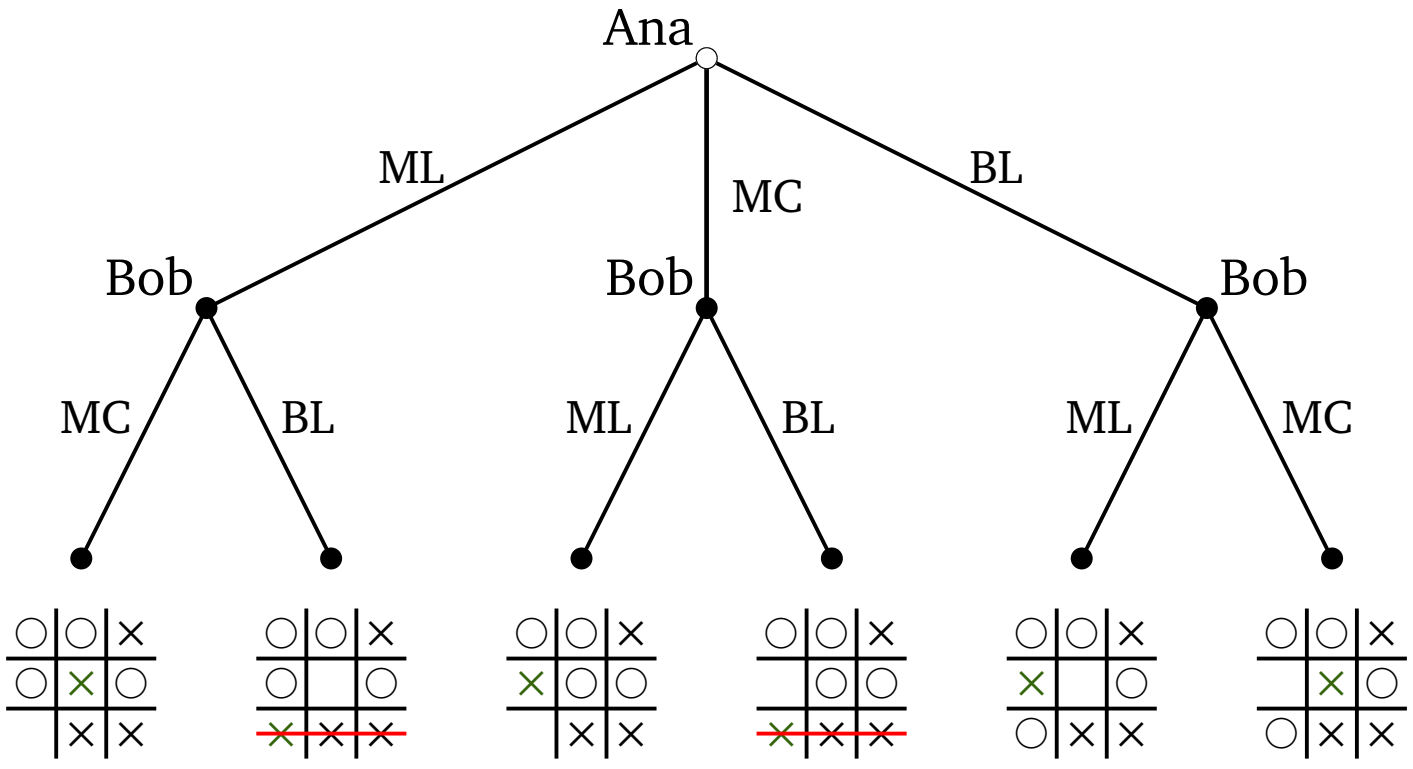
Example: Tic-Tac-Toe

Modeling the end of the game



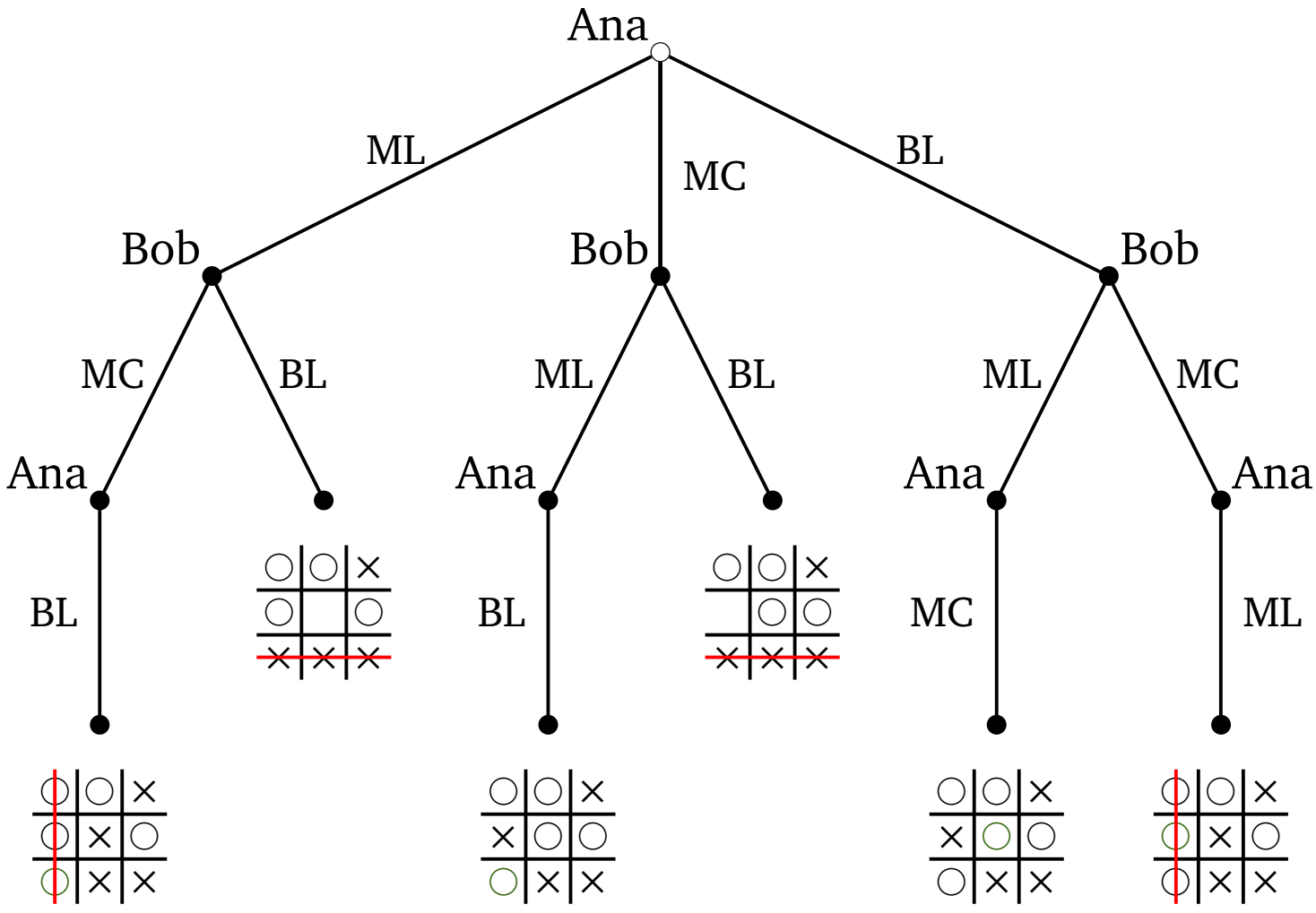
Example: Tic-Tac-Toe

Modeling the end of the game



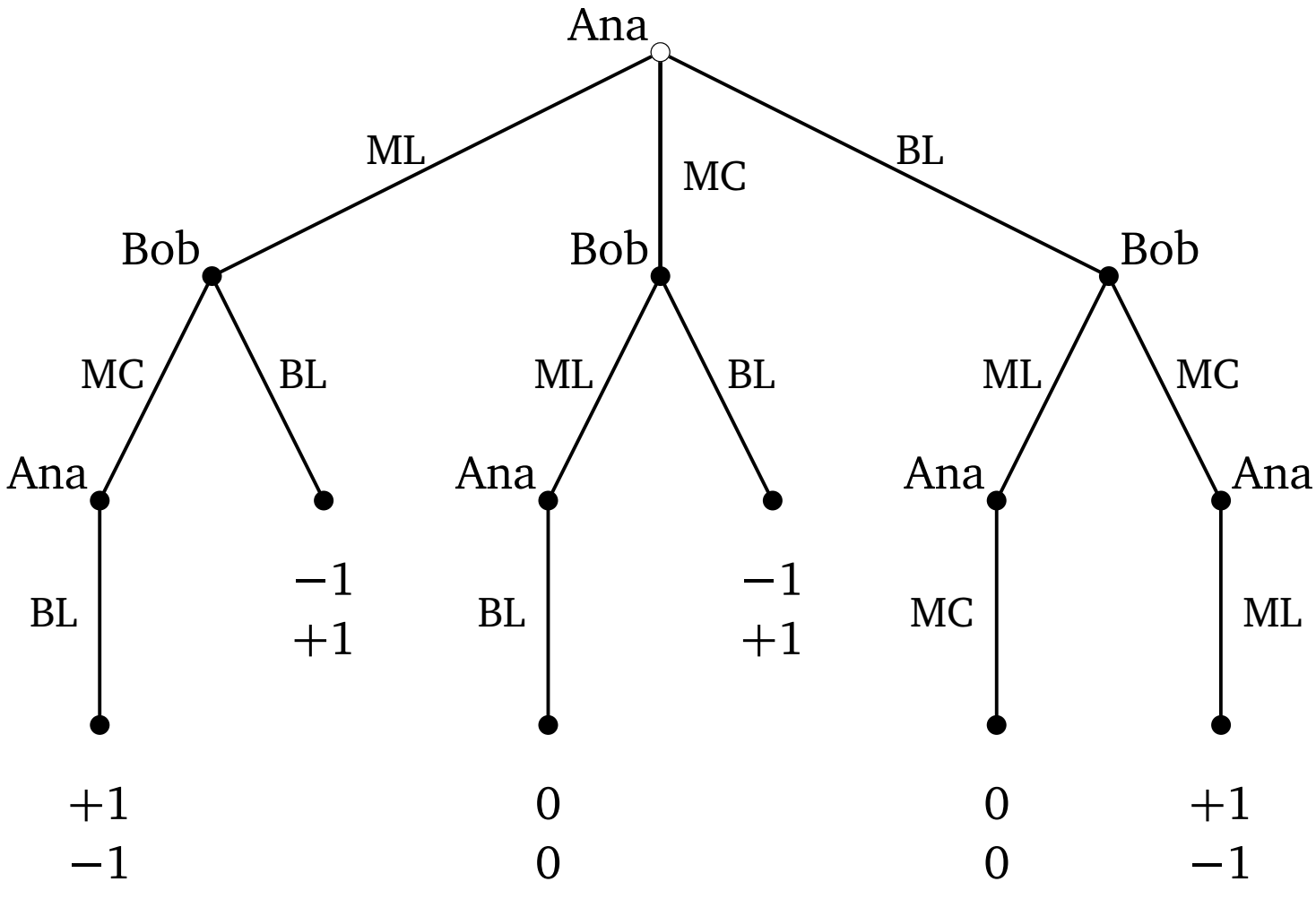
Example: Tic-Tac-Toe

Modeling the end of the game



Example: Tic-Tac-Toe

Modeling the end of the game

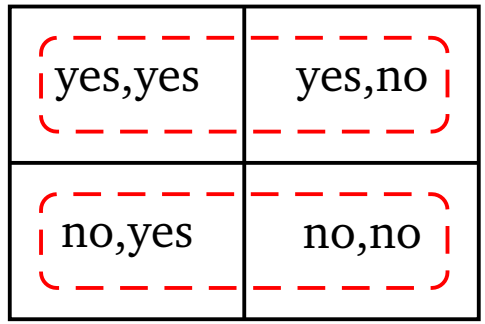


Information

- In the examples covered thus far players always know everything that has happened before their decision
- Games with this property are called perfect information games
- In many situations players have to make decision without knowing the state of the game

Example: Rain and information sets

- Suppose that someone asks you: *Is it raining right now? Will it tomorrow?*
- There are four possible answers (other than I don't know):

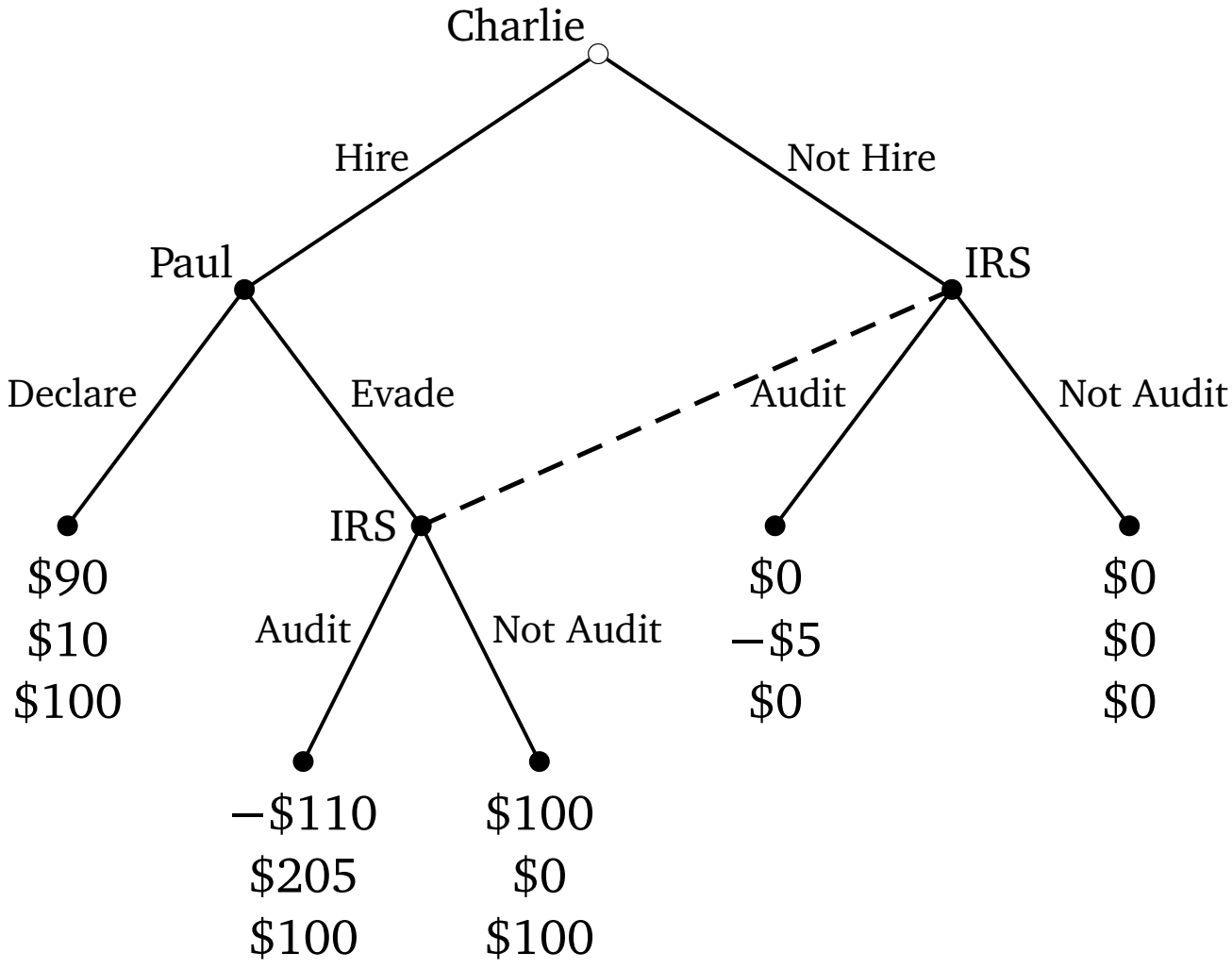


- If it is raining right now you know that the correct answer is either (yes,yes) or (yes,no) but you don't know which
- If it is not raining right now you know that the correct answer is either (no,yes) or (no,no) but you don't know which

Example: collecting taxes

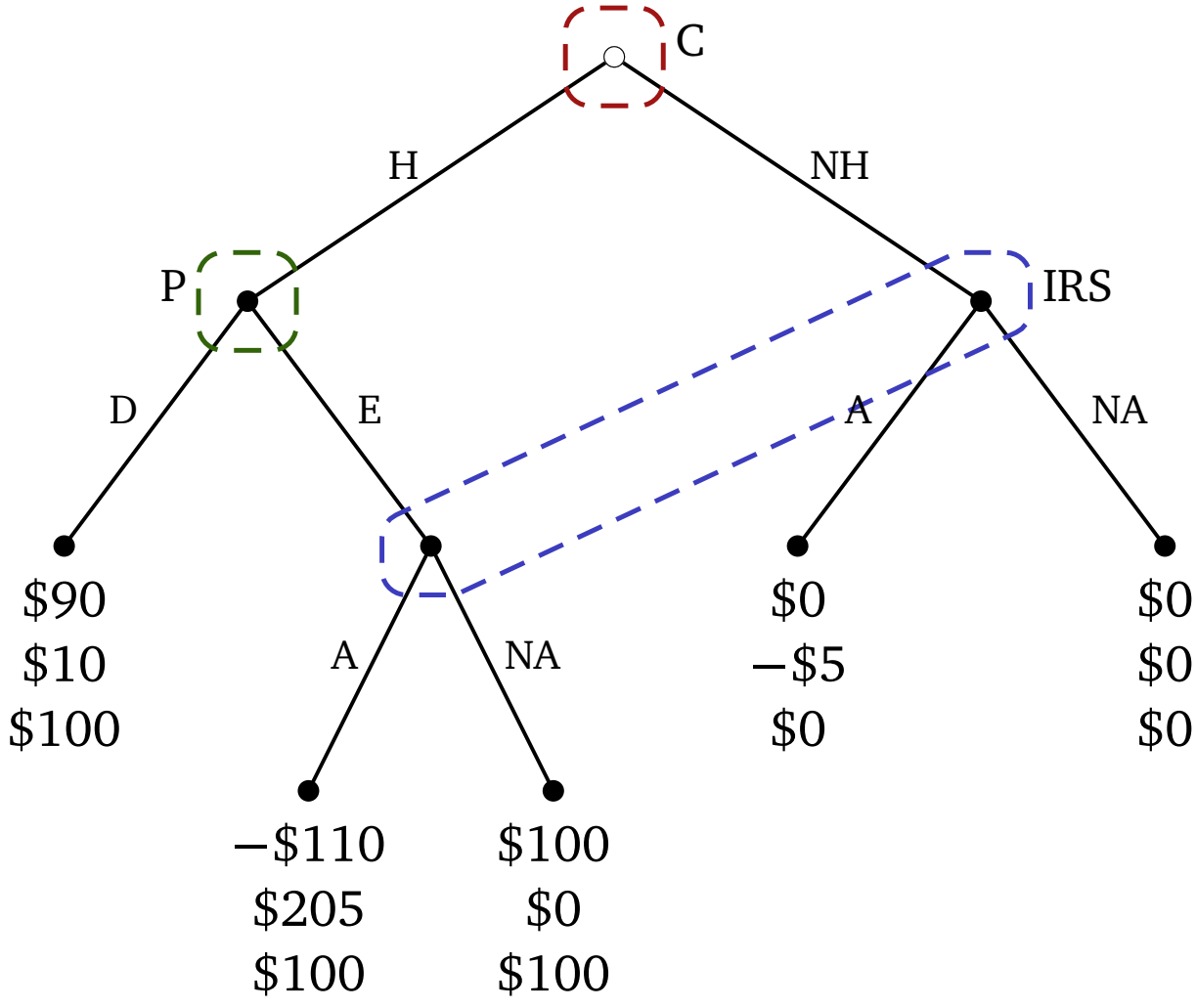
- Paul is a plumber and Charlie is one of his clients
- On a given year she will either hire his services or not. If she hires his services she gets $200 worth of benefits and pays him $100
- If Charlie hires him, Paul has the option to declare the sale to the IRS and pay $10 worth or taxes
- If the IRS receives no declaration they have the option of either audit Paul or not
- In that case, the IRS does not know whether they received no declaration because Charlie didn't hire Paul or because Paul is trying to evade taxes
- The cost of auditing is $5 and, if Paul is caught evading taxes he has to pay a fine of $200 plus the taxes that he should have payed

Example: collecting taxes



Example: collecting taxes

Information sets



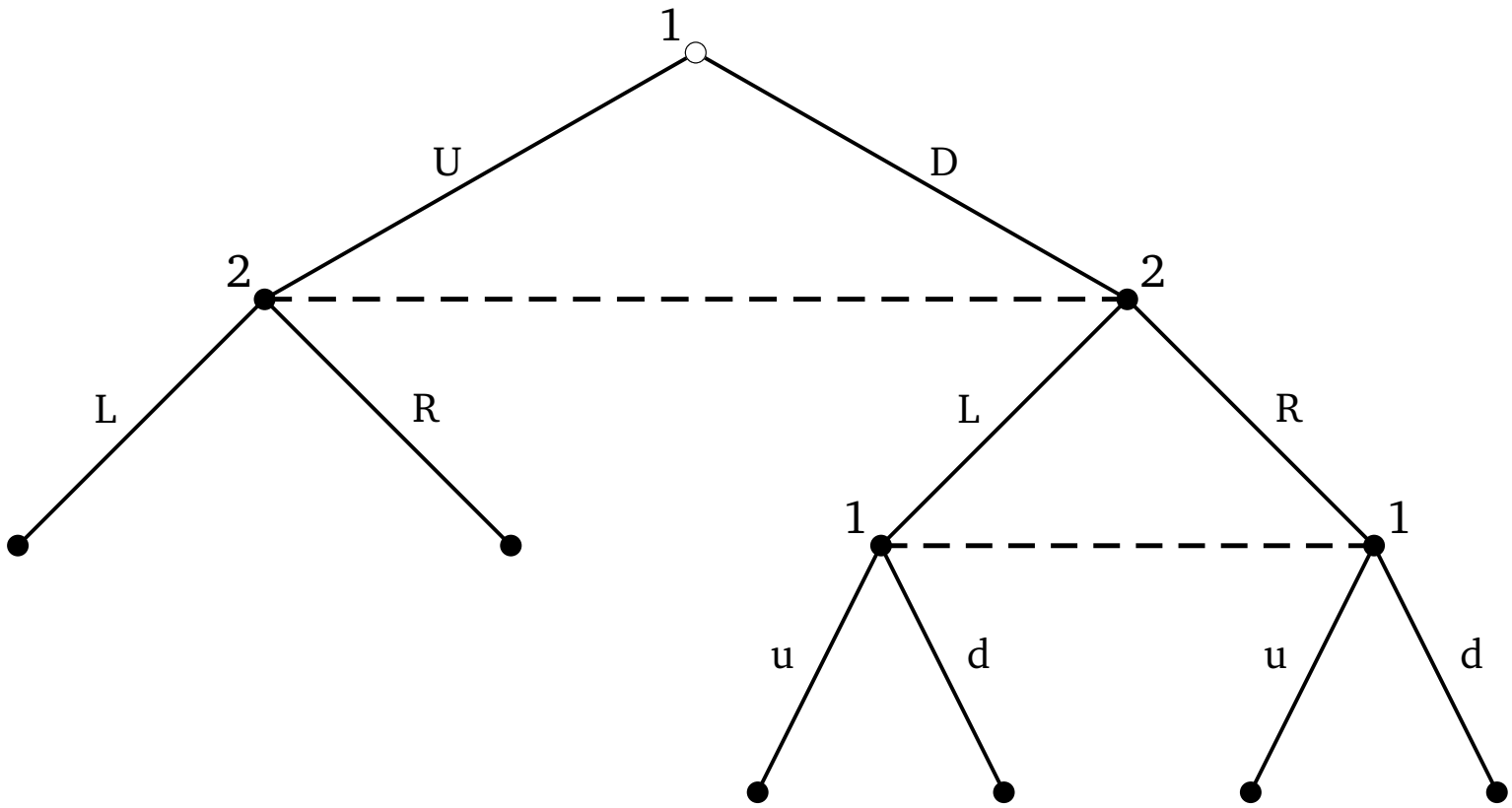
Information sets

Definition

- To capture lack of information in game trees we use information sets
- We group decision nodes that are indistinguishable to the player making a decision at them
- Each group is an information set
- When the game reaches a node in an information set, we assume that the payer knows that the game is in some node within the information set but he/she doesn't know which one
- This implies that a player must behave in the same way in all the nodes within the same information set

Information sets

Example for HW1



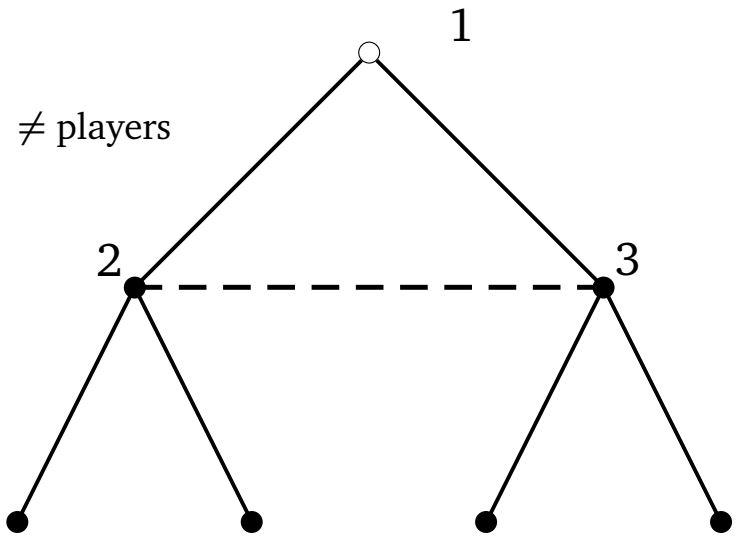
Information set requirements

Optional: Watson §14 pp 159-164

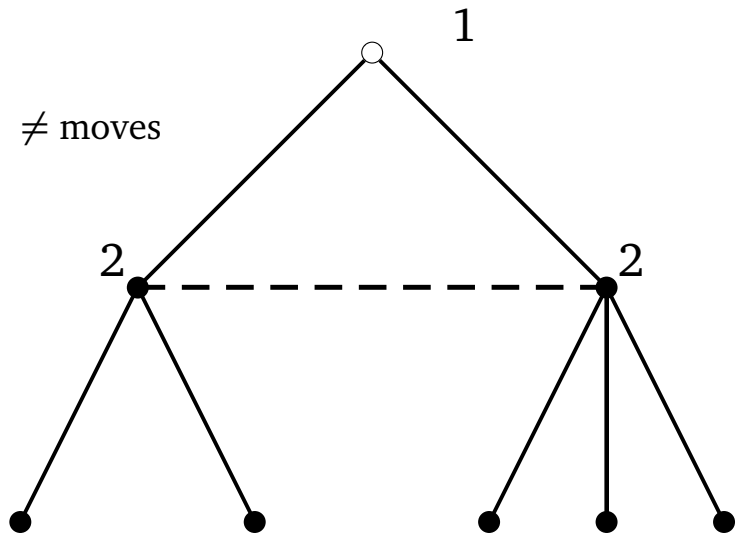
- We will only consider games satisfying three requirements:
  1. Players know when its their turn to make a choice: the same player has to move at all the nodes within *the same* information set
  2. Players know which moves they have available: all the nodes within the same information set must have the same number of outgoing branches
  3. Players never forget any information or any moves they make, this property is called *perfect recall*

Example: **INVALID** information structures

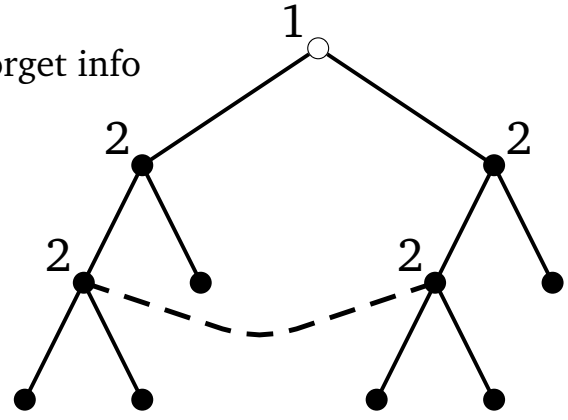
≠ players



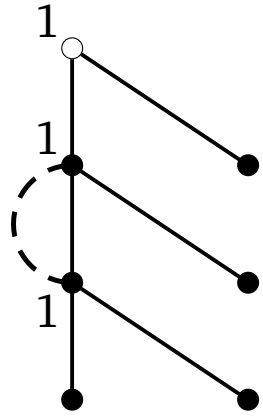
≠ moves



Forget info



Forget move



Simultaneous move games

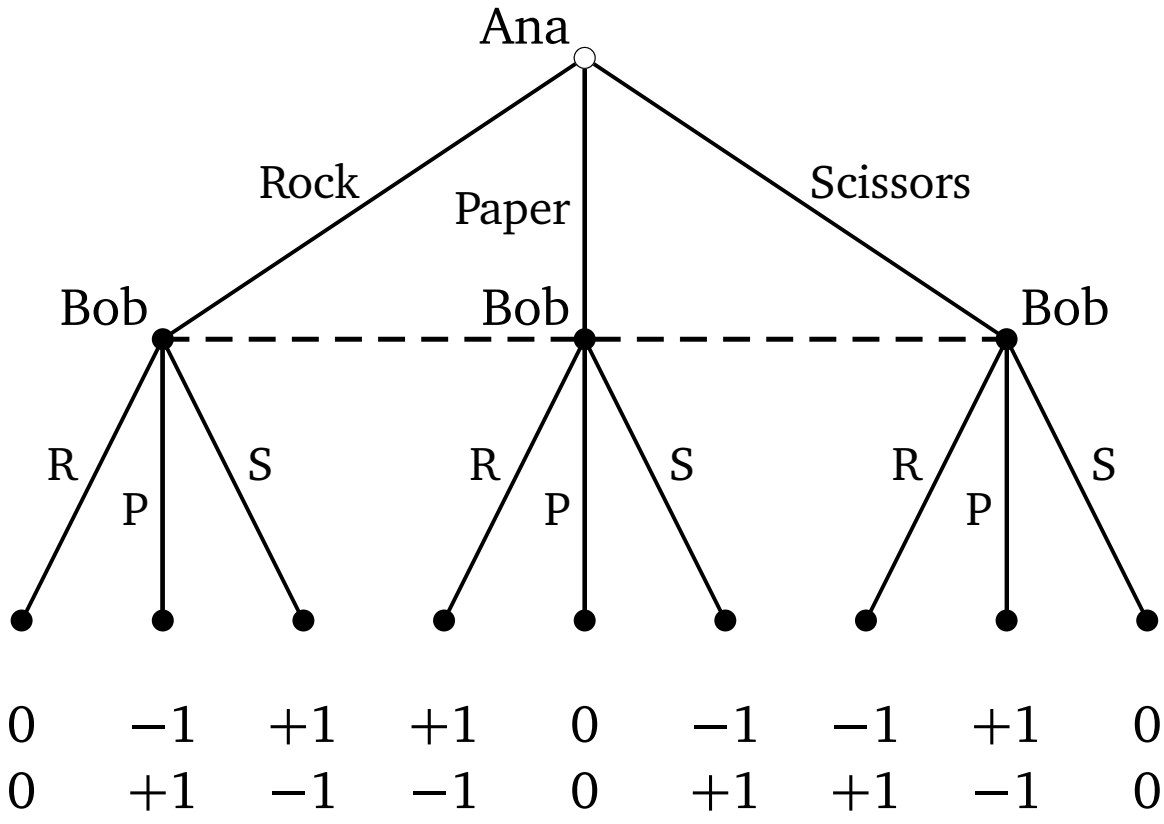
Definition

A simultaneous move game is an extensive form game in which:

1. Each player makes a single choice
2. Each player has no information about his opponent's choices at the moment of making his own

Example: Rock, paper, scissors

Simultaneity and information



Example: Rock, paper, scissors

An equivalent representation