Topics in Informational Economics 1

Games with Incomplete Information and the Principal-Agent Problem

Watson §24-25, pages 291-309

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Games with incomplete information

- Thus far we have studied environments where players know all the relevant information, except perhaps the choices made by their opponents
- We have implicitly assumed that the game being played is common knowledge
- This is rarely the case in real life situations, eg:
  - Poker
  - Choosing a college/mayor
  - Pricing an item
  - Buying a car or a computer
  - Hiring an employee
  - Proposing

Chance moves

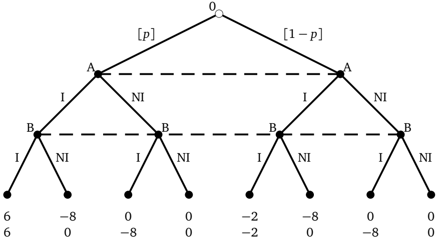
- To allow for incomplete information we include an additional agent that determines the things that are out of the control of the players
- This agent is usually called chance, nature or 0

- Unlike other players, Chance does not have any payoffs
- Instead, we assume that Chance makes choices according to some *commonly known* pre-specified (pure or mixed) strategy

- From the perspective of the players Nature is just another opponent
- All the solution concepts we have studied so far can be directly applied to games with Nature

Example: Risky investment/coordinated attack

- Anna and Bob simultaneously decide whether to invest in a given firm
- If only one of them invests, the firm does not gather enough capital and goes bankrupt
- If both of them invest the firm can make profits or losses depending on the state of the economy *which is unknown for the players*
- The economy is in a good state with probability p and in bad state with probability $1-p$



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	I	NI
I	$6, 6$	$-8, 0$
NI	$0, -8$	$0, 0$

Good (p)

	I	NI
I	$-2, -2$	$-8, 0$
NI	$0, -8$	$0, 0$

Bad $(1 - p)$

Example: Risky investment/coordinated attack

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- If only one of them invests, the firm does not gather enough capital and goes bankrupt
- If both of them invest the firm can make profits or losses depending on the state of the economy *which is unknown for the players*
- The economy is in a good state with probability 1/2 and in bad state with probability 1/2

	I	NI
I	2 , 2	−8 , 0
NI	0 , −8	0 , 0

Principal-agent problems

- Now we will consider environments in which a principal hires agent(s), eg:
  - The owner of a firm hires a manager to run it
  - The manager of a firm hires an employee to work in it
  - A society elects a government official
  - A consumer hires an expert (doctor, mechanic, lawyer, financial advisor) to perform a service

- Principal-agent problems are interesting when the agent and the principal's objectives are not perfectly aligned and:
  1. The decisions of the agent are not contractible, eg you can't verify in court the effort provided by an employee
  2. The agent is better informed than the principal, eg your doctor knows which treatment is better for you

- In such cases, there might not exist an efficient contract
- We will only consider the first kind of issues (non-contractible choices)

Principal-agent problems

- We assume that the principal has all the bargaining power: he/she offers a contract and then the agent decides whether to accept it and, if he/she accepts it performs the corresponding services
- We consider contracts that specify what the agent(s) should do and a transfer rule that specifies the payment that the agent receives conditional on *contractible* outcomes

- There are two requirements for a contract to be valid:

  1. *Individual rationality*.– The agent(s) should be willing to accept the contract, ie the contract should offer the agent the possibility of getting at least his/her outside option
  2. *Incentive compatibility*.– The agent(s) should be willing to do what the contract tells them to do, ie the instructions in the contract should induce a SPNE of the resulting game

- We want to find an incentive compatible and individually rational contract that maximizes the principal's payoff

Example: Providing effort

- Anna wishes to hire Bob to work in a project
- If hired, Bob will choose whether to provide high effort (H) or low effort (L) and he will receive a monetary transfer T from Anna that is contingent on the realized outcome
- The cost of effort for Bob is $C(L) = 0$, $C(H) = 1$
- The revenue of the enterprise depends both on bob's effort:
  - If Bob provides a high level of effort the revenue is $\pi^H = 20$ with probability 3/4 and $\pi^L = 4$ with probability 1/4, yielding an expected revenue of 16
  - If Bob provides a low level effort the revenue is π^L for sure
- Anna's payoff is the revenue of the firm minus whatever she pays to Bob: $u_A = \pi - T$
- Bob's payoff if he rejects the contract is 1 and, if he accepts the contract it is $u_B = \sqrt{T - C}$ (risk aversion)

Example: Providing effort

Observable effort

- Anna would like Bob to provide high effort as long as this costs her less that $16 - 4 = 12$
- If effort where observable Anna could offer a contract (S, ω) that promises to pay a base wage ω plus a bonus b that will be paid *only if he provides high effort*
- For Bob to provide high effort (IC) it must be the case that:

$$u_B(H|\omega, b) = \sqrt{\omega + b - 1} \geq \sqrt{\omega} = u_L(H|\omega, b) \qquad \Leftrightarrow \qquad b \geq 1$$

- For Bob to accept this contract (IR) it must be the case that he gets at least his outside option, i.e.

$$\sqrt{\omega + b - 1} \geq 1 \quad \Leftrightarrow \quad \omega + b \geq 2$$

- Since $\omega + b$ is the total transfer that Anna will pay to Bob, any contract with $\omega + b = 2$ and $b \geq 1$ is optimal
- The optimal profit for Anna is $u_A^* = 16 - 2 = 14$

Example: Providing effort

Non-observable effort

- Now suppose that effort is not contractible, transfers can be contingent only on the total firm revenue
- Anna can offer a base wage ω plus a bonus b *contingent on a high revenues*
- In this case the IC constraint is:

$$u(H|\omega, b) = \frac{3}{4}\sqrt{\omega + b - 1} + \frac{1}{4}\sqrt{\omega - 1} \geq \sqrt{\omega} = u(L|\omega, b)$$

- The IR constraint is:

$$\frac{3}{4}\sqrt{\omega + b - 1} + \frac{1}{4}\sqrt{\omega - 1} \geq 1$$

- As before, an optimal contract will result from satisfying both constraints with equality which implies

$$\sqrt{\omega} = 1 \quad \Rightarrow \quad \omega = 1 \quad \Rightarrow \quad \frac{3}{4}\sqrt{b} = 1 \quad \Rightarrow \quad b = \frac{16}{9}$$

- In this case, Anna's expected payoff is:

$$u_A^* = \frac{3}{4}\left(20 - 1 - \frac{16}{9}\right) + \frac{1}{4}\left(2 - 1\right) = \frac{158}{12} \approx 13.16$$

Example: Performance measures

Description

- Now suppose that Bob can decide how much effort to provide for two different tasks, let $x, y \in [0, 10]$ be the effort provided for each task
- As before, suppose that effort is not contractible but there is a contractible objective performance measure
- Suppose that:

$$\pi(x, y) = 2x + y + \psi$$
$$p(x, y) = y + 2x + \varepsilon$$
$$c(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$$

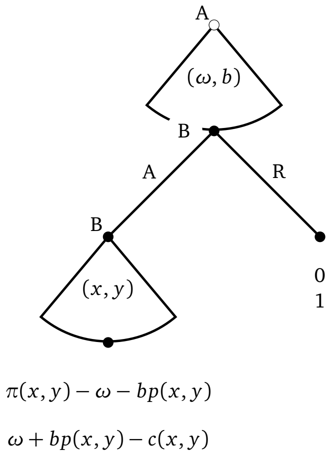
  item where $\psi, \varepsilon \sim N(0, 1)$ are independent random variables
- Anna can offer a contract (ω, b) and the total transfer made is:

$$T(x, y|\omega, b) = \omega + b \cdot p(x, y)$$

- Anna's payoff is $\pi - T$, Bob's payoff is $T - c$ and Bob's outside payoff is 1

Example: Performance measures

EFG



Example: Performance measures

Backward induction

- If Bob accepts the contract he will choose x, y to maximize:

$$U_B(x, y|\omega, b) = by + 2bx - \frac{1}{2}x^2 - \frac{1}{2}y^2$$

- The optimal choices are $y^* = b \qquad x^* = 2b$
- Doing backward induction, Anna chooses b to maximize:

$$\begin{aligned} U_A(b|\omega, x^*, y^*) &= 2y^* + x^* - b(y^* + 2x^*) - \omega \\ &= 4b - 5b^2 - \omega \end{aligned}$$

- This implies that $b^* = 4/10$ and thus $x^* = 8/10$ and $y^* = 4/10$
- ω^* is determined by the IR constraint:

$$\begin{aligned} 1 \leq U_B(x^*, y^*|\omega^*, b^*) = \omega^* + b^*(y^* + 2x^*) &= \omega^* + \frac{4}{10}\left(\frac{4}{10} + 2\frac{8}{10}\right) \\ &= \omega^* + \frac{8}{10} \quad \Rightarrow \quad \omega^* = \frac{2}{10} \end{aligned}$$

Example: Performance measures

Efficiency loss

- In equilibrium Bob gets exactly his outside option $U_B = 1$ and Anna gets:

$$U_A^* = \pi^* - T^* = 2\frac{4}{10} + \frac{8}{10} - 1 = \frac{6}{10}$$

- In contrast, efficiency requires maximizing:

$$\pi(x,y) - c(x,y) = 2y + x - \frac{1}{2}y^2 - \frac{1}{2}x^2$$

- Which implies that the efficient effort levels are $x^E = 1$ and $y^E = 2$
- If effort where observable, Anna could pay Bob 1.5 conditional on him providing the optimal level of effort and payoffs would be:

$$U_B = 1.5 \qquad U_A = 5 - 1.5 = 3$$

- Inefficiency will prevail as long as the performance measure is not perfectly aligned with the revenue function