Modeling Strategic Environments 2

Strategic form games

Watson §3, pages 24-35

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Strategies

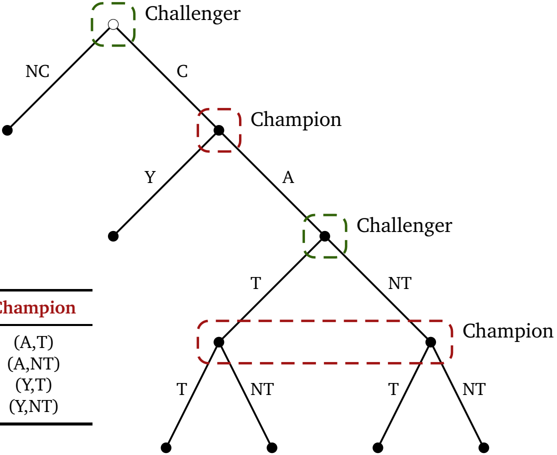
Definition

A strategy is a *complete contingent* plant for a player in a game

- A strategy must specify a choice at every possible decision point
- Since players must behave the same way on different nodes within the same information set, a strategy specifies a choice at every information set
- The adjective "every" is very important, it means every decision point, even those that won't be reached!!
- We think of a strategy as an instructions manual. A player could give this manual to a machine and the machine would know what to do under *every possible contingency*

Example: Heavyweight championship

Extensive form and strategies



Challenger
(C,T)
(C,NT)
(NC,T)
(NC,NT)

Champion
(A,T)
(A,NT)
(Y,T)
(Y,NT)

Strategic form games

- Any possible way of playing the game can be captured by a strategy
- If we know the strategy being followed by each player we can determine the path of play and the outcome
- This suggests a simplified form of representing the strategic environment

Definition

A strategic form game is a mathematical object that specifies:

1. The set of players
2. The set of strategies available to each player
3. A function that specifies the payoff that each player will receive for each strategy profile

Example: Heavyweight championship

Strategic form game

Champion

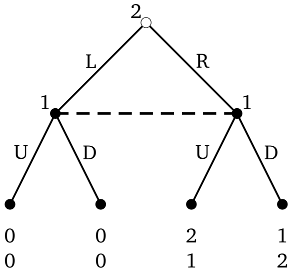
Challenger

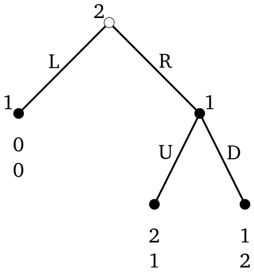
	(A,T)	(A,NT)	(Y,T)	(Y,NT)
(C,T)	-1 , 4	-3 , 4	3 , -2	3 , -2
(C,NT)	4 , -3	5 , -3	3 , -2	3 , -2
(NC,T)	0 , 0	0 , 0	0 , 0	0 , 0
(NC,NT)	0 , 0	0 , 0	0 , 0	0 , 0

Equivalent representations

- An extensive form game is a more detailed description of the strategic environment
- When we switch to a strategic form game we lose some information
- As a result:
  - An extensive form game admits a unique strategic form representation
  - A strategic form game represents different extensive form games
- It is commonly argued that strategic form games contain all the *strategically relevant* information, at least in most situations

Example: Equivalent representations





	L	R
U	0 , 0	2 , 1
D	0 , 0	1 , 2

Notation for strategic form games

- i denotes a generic player and $-i$ denote the set of his/her opponents
- S_i denotes the set of strategies available for player i and we denote a generic strategy with s_i
- $S = \times_i S_i$ denotes the set of strategy profiles, i.e. vectors that specify a strategy for each player
- s denotes a generic strategy profile
- $u_i(s)$ denotes the corresponding payoff for player i
- Given a strategy profile $s = (s_1, s_2, \ldots, s_N)$ we use the (unfortunate) notation $s = (s_i, s_{-i})$ with $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_{N-1}, s_N)$ a vector that specifies a strategy for everyone except i
- **This notation is meant to help you**. You will not need to use it in the exams or homework but it is necessary to properly define some of the concepts we will use.

Strategic form games

Formal definition

A strategic form game is a mathematical object consisting of :

1. A set of N players indexed by $i \in I = \{1, 2, \ldots, N\}$
2. A set of strategies S_i for each player $i \in I$
3. A function $u_i : \times_i S_i \to \mathbb{R}$ for each player $i \in I$ that represents his/her payoff for each strategy profile

Example: Prisoner's dilemma

- Two suspects of a crime are arrested
- The DA has enough information to convict them for a misdemeanor (1 year in prison) but can increase their sentence if she obtains a confession
- Both prisoners are offered a sentence reduction in exchange for a confession:
  - If one prisoner confesses he walks free but his accomplice is sentenced to 5 years in prison.
  - If both prisoners confess they are sentenced to 3 years in prison each

	Keep Silent	Confess
Keep silent	$-1, -1$	$-5, 0$
Confess	$0, -5$	$-3, -3$

Example: Prisoner's dilemma

Alternative interpretation

- A "closed bag" barter is going to take place
- Each party values his object 2 and his opponent's object 3
- Each party can choose to fill the bag or not

	Full	Empty
Full	3 , 3	0 , 5
Empty	5 , 0	2 , 2

Example: Meeting in NY

- Daniel is travelling to NY to meet with Charlie
- Charlie was supposed to pick up Daniel at the train station but they forgot to specify which and they have no way of communicating with each other (old example)
- They both have to choose between Grand Central Station or Penn Station

	GCS	PS
GCS	1 , 1	0 , 0
PS	0 , 0	1 , 1

Example: Battle of the Sexes

- Mike and Nancy want to go on a date
- Mike wants to go to a football game while Nancy prefers the opera
- They both prefer going to a place they don't like over not having a date at all

	Football	Opera
Football	5 , 1	0 , 0
Opera	0 , 0	1 , 5

Example: Chicken

- Based on “Rebel without a cause” y2u.be/u7hZ9jKrwvo
- Players drive towards each other (or towards a cliff) they can continue driving straight or they can swerve to avoid a crash
- If only one player swerves he/she is a “chicken” which is something shameful but better than crashing and dying

	Continue	Swerve
Continue	0 , 0	5 , 1
Swerve	1 , 5	2 , 2

Example: Pigs

- There is a strong but slow pig and a weak but fast piglet
- They have to push a button in order to get some food
- The button is far away from the den where the food is dispensed
- Once the pig gets to the food, the piglet is pushed away and won't get to eat anything else
- The piglet only gets to eat if he gets to the food before the pig

		Fast: Press	Fast: Don't press
Strong	Press	$3, 1$	$0, 5$
	Don't press	$6, -2$	$-1, -1$

Example: Matching Pennies

- Two kids secretly place a penny in their hand with either heads or tails facing up
- They reveal their pennies simultaneously, if they pennies match the fist kid wins and if they differ the second kid wins

	Heads	Tails
Heads	$-1\,,+1$	$+1\,,-1$
Tails	$+1\,,-1$	$-1\,,+1$

Example: Rock, Paper, Scissors

	Rock	Paper	Scissors
Rock	$0\,,\,0$	$-1\,,\,+1$	$+1\,,\,-1$
Paper	$+1\,,\,-1$	$0\,,\,0$	$-1\,,\,+1$
Scissors	$-1\,,\,+1$	$+1\,,\,-1$	$0\,,\,0$

Example: uneven thumb

- Three kids simultaneously reveal a thumb pointing either up or down
- If all thumbs point in the same direction the game is a draw
- Otherwise the kid with the uneven thumb looses

	Up	Down
Up	$0, 0, 0$	$1, -1, 1$
Down	$-1, 1, 1$	$1, 1, -1$

Up

	Up	Down
Up	$1, 1, -1$	$-1, 1, 1$
Down	$1, -1, 1$	$0, 0, 0$

Down

Example: Cournot competition

- Three firms indexed by 1, 2 and 3 sell the same commodity
- Firms simultaneously choose quantities in $[0, 100]$. Let x be the quantity chosen by firm 1, y be the quantity chosen by firm 2 and z be the quantity chosen by firm 3
- The price is determined by the market according to the inverse demand function:

$$p(x, y, z) = 100 - x - y - z$$

- Firms have constant marginal cost equal to 2 so that their profits are given by:

$$u_1(x, y, z) = \Big(p(x, y, z) - 2\Big)x = -x^2 + (100 - y - z)x$$
$$u_2(x, y, z) = \Big(p(x, y, z) - 2\Big)y = -y^2 + (100 - x - z)y$$
$$u_3(x, y, z) = \Big(p(x, y, z) - 2\Big)z = -z^2 + (100 - x - y)z$$

Example: Bertrand competition

- Two firms indexed by 1 and 2 sell commodities that are imperfect substitutes
- Firms choose prices in $[0, 10]$ simultaneously and independently. Let p be the price chosen by firm 1 and q be the price chosen by firm 2.
- The quantity demanded for each commodity depends on the prices of both firms and is given by:

$$D_1(p,q) = 10 - p + \frac{1}{2}q \qquad D_2(p,q) = 10 - q + \frac{1}{2}p$$

- Firms have constant marginal cost equal to 2 so that their profits are given by:

$$u_1(p,q) = \left(p-2\right)D_1(p,q) = -p^2 + \left(12 + \frac{1}{2}q\right)p - \left(20 + q\right)$$

$$u_2(p,q) = \left(q-2\right)D_2(p,q) = -q^2 + \left(12 + \frac{1}{2}p\right)q - \left(20 + p\right)$$