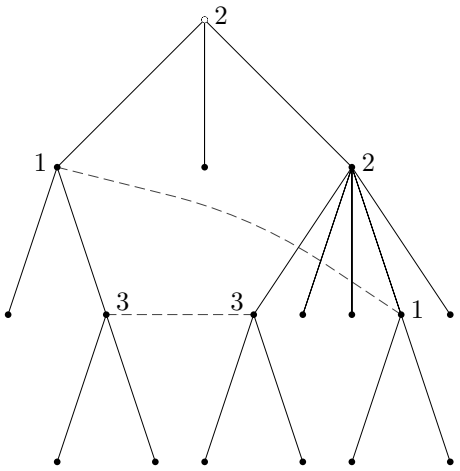
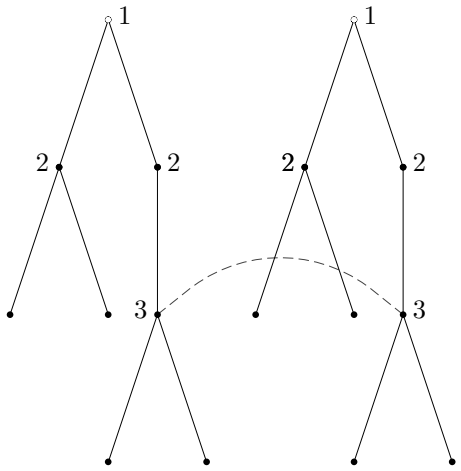
Econ 4020 – Problem Set I

Due on 02/23

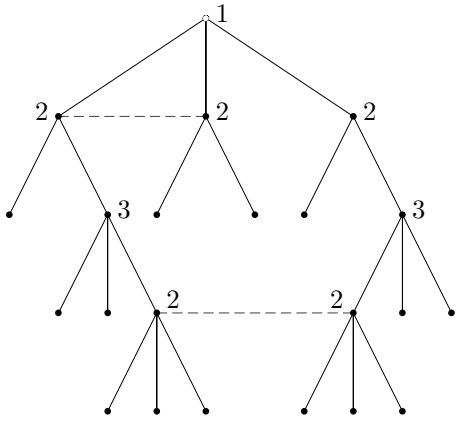
1. Determine which of the following are valid game trees with valid information structures. For the ones that are invalid, explain why.



(a)



(b)



(c)

2. Susan and Peter are neighbors and are both getting ready to attend an event. Each of them has to choose between wearing red and wearing blue. Susan prefers to wear a red outfit and Peter prefers to wear blue. Each receives additional utility of 1 from wearing his or her preferred color. In addition, Peter wants to match Susan and he gets an additional utility of 2 from matching. Meanwhile, Susan

does *not* want to match Peter and gets a disutility of -1 if they match. The timing is as follows. Susan chooses her outfit first and leaves the house. She can choose to walk out the front door so that Peter can see which outfit she is wearing, or leave through the back door in which case Peter cannot see anything. After this, Peter chooses his outfit.

(a) Write down an extensive form game representing this situation.

(b) Write down a strategic form game representing this situation.

(c) Write a different extensive form game with the same strategic form.

3. Anna is considering buying wine from Bob for \$100. The wine is good with probability 30% and bad with probability 70%. Ana's utility from consuming wine is \$200 if it is good and \$20 if it is bad. After tasting the wine to determine its quality, Bob has the option to send a message to Anna stating that the wine is good, send a message saying that the wine is bad, or not sending a message at all. Talk is cheap: there is no cost for Bob to send a message, and he can lie if he wants to (but he can send at most one message). Anna observes the message but not the quality of the wine before choosing to buy it or not.

(a) Write down an extensive form game representing this situation.

(b) Now suppose that Bob is a terrible sommelier and cannot tell good wine from bad wine. He learns nothing from the tasting, but still can send a message to Anna. Write down an extensive form game representing this situation.

(c) How many strategies does each player have in each case?

4. David's utility for money is given by $u(x) = x^\alpha$, where $\alpha = 1/2$ is a parameter. David's current wealth is $\omega = \$9$. He is offered to make a bet on the outcome of Cornell's next football game. He will get paid \$7 if he wins the bet, and will have to pay \$9 if he loses the bet. He can either bet that Cornell will win, or bet that Cornell will lose. He believes that Cornell will win with probability $p \in (0, 1)$.

(a) Write down an expression for David's expected utility for betting for Cornell, betting against Cornell, and for not betting at all.

(b) Which actions are David's best responses as a function of p?

(c) Which actions are rational (i.e., a best response to *some* belief)?

(d) What happens to the set of beliefs for which *not* betting is optimal when α increases?

(e) What happens to the set of beliefs for which *not* betting is optimal when ω increases? (suppose David still loses \$9 if he loses the bet)

5. Consider the following game in strategic form

	a	b	c	d
w	0 , 7	2 , 5	7 , 0	0 , 1
x	5 , 2	3 , 3	5 , 2	0 , 1
y	7 , 0	2 , 5	0 , 7	0 , 1
z	0 , 0	0 , −2	0 , 0	10 , −1

(a) Which actions are strictly dominated for each player?

(b) Is there any belief for which strategy d is a best response?

(c) Is there any belief for which strategy z is a best response?

(d) Find all rationalizable strategies for each player.

6. Anna and Bob work as partners. The firm's revenue depends on the level of effort provided by each of them. Each of them can provide any level of effort in $[0, 5]$. Let A denote the level of effort provided by Anna, and B the level of effort provided by Bob. Providing effort is costly. The cost for Anna is $-A^2$ and the cost for Bob is $-B^2$. The total revenue of the firm equals $4A + 4B + 2AB$. Anna and Bob receive half the firm's revenue each.

(a) Write down a strategic form game representing this situation. Who are the players? What strategies do they have available? What are the payoff functions?

(b) Find an analytic solution for the best response functions and graph them in a clearly labeled figure.

(c) Can Anna rationalize choosing $A = 4$? How about $A = 2.5$ or $A = 1.5$? Justify your answer *in detail*.

(d) Find a level of effort that is rationalizable for Anna.