Econ 4020 – Second Preliminary Exam Practice

There are 6 problems. You have 70 minutes. Justify all your answers. Good luck!

1. What is your name?

2. What percentage grade from 0 to 100 do you think you will get on this exam?

3. Find all the NE, both in pure and mixed strategies, for the following game The

	a	b	c
x	1 , $\underline{7}$	1 , 5	$\underline{3}$, 4
y	$\underline{2}$, 3	0 , 4	0 , $\underline{6}$

unique NE has $\sigma_1^* = (1/2, 1/2)$ and $\sigma_2^* = (3/4, 0/1/4)$.

Justification: There are no pure NE and b is dominated by $0.5a + 0.5c$. Let $\Pr(a) = q$ and $\Pr(c) = 1 - q$. For player 1

$$U_1(x; q) = q + 3(1-q) = 3 - 2q \quad \& \quad U_1(y; q) = 2q.$$

Hence, for player 1 to be indifferent we need

$$2q = 3 - 2q \qquad \Rightarrow \qquad q = \frac{3}{4}.$$

The for finding $\sigma_1^*(x)$ is analogous.

4. Anna and Bob bargain to split \$100 following the protocol described as follows. There are at most two rounds, and players do *not* discount the future ($\delta = 1$). On each round, a player is selected at random to act as the proposer. Anna is selected with probability $p \in (0, 1)$ and Bob is selected with probability $1 - p$. The proposer proposes a split $(x, 100 - x)$ with $0 \leq x \leq 100$. The other player either accepts or rejects the proposal. If the offer is accepted, the game ends with payoffs $(x, 100 - x)$. If an offer is rejected on the first round, the game moves onto the second round. If an offer is rejected on the second round, the game ends with payoffs $(0, 0)$.

(a) Find a SPNE of the game

- On the second period:
  - Anna would accept any offer, and she would offer the split $(100, 0)$

- Bob would accept any offer, and he would offer the split $(0, 100)$
- On the first period
  - Anna would accept a split $(x, 100 - x)$ if and only if $x \geq 100p$, and she would offer the split $(100p, 100(1 - p))$
  - Bob would accept a split $(x, 100-x)$ if and only if $100-x \geq 100(1-p)$, and he would offer the split $(100p, 100(1 - p))$

Justification: The second period subgame after a proposer is selected is just an ultimatum bargaining game like the one we discussed in class. If the game reaches the second round, Anna's payoff would be 100 if she is chose, and 0 otherwise. Hence, her expected utility would be $100p$. Similarly, Bob's expected utility on the second period would be $100(1-p)$. (Why is does this show that the proposed strategies are a SPNE?).

(b) Is the SPNE unique? (Justify your answer) No. The following strategy profile is also a SPNE (why?)

- On the second period:
  - Anna would accept any offer, and she would offer the split $(100, 0)$
  - Bob would accept any offer, and he would offer the split $(0, 100)$
- On the first period
  - Anna would accept a split $(x, 100 - x)$ if and only if $x > 100p$, and she would offer the split $(100p, 100(1 - p))$
  - Bob would accept a split $(x, 100-x)$ if and only if $100-x > 100(1-p)$, and he would offer the split $(100p, 100(1 - p))$

5. Anna and Bob work as partners. The firm's revenue depends on the level of effort provided by each of them. Each of them can provide any level of effort in $[0, 100]$. Let A denote the level of effort provided by Anna, and B the level of effort provided by Bob. Providing effort is costly. The cost for Anna is $-A^2$ and the cost for Bob is $-2B^2$ (note that the game is *not* symmetric). The total revenue of the firm equals $A + B + AB$. Anna and Bob receive half the firm's revenue each. For Anna:

$$
\begin{aligned}
& u_A = \frac{1}{2}(A + B + AB) - A^2 \\
\Rightarrow \quad & u'_A = \frac{1}{2}(1 + B) - 2A \\
\Rightarrow \quad & \mathrm{BR}_A(B) = \frac{1}{4}(1 + B)
\end{aligned}
$$

Similarly for Bob:

$$\text{BR}_B(A) = \frac{1}{8}(1 + A)$$

(a) Find the unique SPNE assuming that Anna and Bob choose their levels of effort independently. A SPNE of the simultaneous move game (A^*, B^*) must satisfy $A^* = \text{BR}_A(B^*)$ and $B^* = \text{BR}_B(A^*)$. Hence we must have:

$$\begin{aligned} & A^* = \text{BR}_A(\text{BR}_B(A^*)) = \frac{1}{4}\left(1 + \frac{1}{8}(1 + A^*)\right) \\ \Rightarrow \quad & 32A^* = 8 + (1 + A^*) \\ \Rightarrow \quad & A^* = \frac{9}{31} \end{aligned}$$

and

$$B^* = \text{BR}_B\left(\frac{9}{31}\right) = \frac{1}{8}\left(1 + \frac{9}{31}\right) = \frac{5}{31}$$

(b) Find the unique SPNE assuming that Anna chooses her level of effort first, and then Bob chooses his level of effort after observing Anna's level of effort. After observing Anna's effor, Bob will choose $B = \text{BR}(A)$. Knowing this, Anna will maximize

$$\begin{aligned} u_A &= \frac{1}{2}(A + \text{BR}_B(A) + A\text{BR}_B(A)) - A^2 \\ &= \frac{1}{2}\left(A + \frac{1}{8}(1 + A) + \frac{1}{8}(1 + A)A\right) - A^2 = \frac{1}{16} + \frac{5}{8}A - \frac{15}{16}A^2 \end{aligned}$$

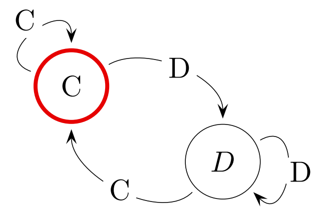
This is a concave function. The first order condition for a maximum is

$$u'_A = \frac{5}{8} - \frac{15}{32}A^* = 0 \quad \Rightarrow \quad A^* = \frac{5}{8} \times \frac{32}{15} = \frac{4}{3}$$

6. Consider an infinitely repeated prisoners dilemma with $\delta = 0.9$ and stage game payoffs given in the table below, and the tit-for-tat strategy described as follows:

- Cooperate on the first period
- On each period after the first one $t > 1$, take the same action your opponent took on period $t - 1$

	C	D
C	$9,9$	$0,10$
D	$10,0$	$1,1$



(a) Suppose that the outcome today is (C, D) and players are using tit-for-tat strategies, what would be the expected discounted continuation payoff? The sequence of continuation outcomes would be:

$$(D, C), (C, D), (D, C), (C, D), \ldots$$

The sequence of per-period payoffs starting on $t+1$ would be

$$(10, 0), (0, 10), (10, 0), (0, 10), (10, 0), \ldots$$

The continuation value for player 1 would be:

$$\begin{aligned} w_1 &= 10 + \delta 0 + \delta^2 10 + \delta^3 0 + \delta^4 10 + \ldots \\ &= 10 + \delta^2 10 + \left(\delta^2\right)^2 10 + \left(\delta^2\right)^3 10 + \ldots = \frac{1}{1-(\delta^2)} 10 = \frac{1}{0.19} 10 = \frac{1000}{19} \end{aligned}$$

The continuation value for player 2 would be (why?)

$$w_2 = \delta w_1 = \frac{900}{19}$$

(b) Do the tit-for-tat strategies constitute a NE of the supergame? If the outcome today is (C, C) and players are using tit-for-tat strategies, then the continuation values are (why?)

$$w_i(C, C) = \frac{1}{1-\delta} 9 = 90$$

Similarly, $w_i(D, D) = 10$. $w_i(C, D)$ and $w_i(D, C)$ follow form part (a). Hence, the incentives along the equilibrium path are captured by the table:

$$\underset{v}{\begin{array}{c|c|c|} & C & D \\ \hline C & 90, 90 & \frac{900}{19}, \frac{1000}{19} \\ \hline D & \frac{1000}{19}, \frac{900}{19} & 10, 10 \\ \hline \end{array}} = \underset{u}{\begin{array}{c|c|c|} & C & D \\ \hline C & 9, 9 & 0, 10 \\ \hline D & 10, 0 & 1, 1 \\ \hline \end{array}} + \delta \underset{w}{\begin{array}{c|c|c|} & C & D \\ \hline C & 90, 90 & \frac{1000}{19}, \frac{900}{19} \\ \hline D & \frac{900}{19}, \frac{1000}{19} & 10, 10 \\ \hline \end{array}}$$

This is true *regardless* of the past history of the game. Along the equilibrium path, the tit-for-tat strategies always prescribe cooperation. Since, (C, C) is a NE of the strategic form game with payoffs v, it follows that both agents using tit-for-tat is a NE of the supergame.

(c) Do the tit-for-tat strategies constitute a SPNE of the supergame? No.

Justification: Tit-for-tat strategies prescribe defection at some histories (off the equilibrium path). However, defection is dominated on the strategic form game with payoffs v.