Strategic Form Games

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Reading assignments: Watson, Ch. 3 & 4

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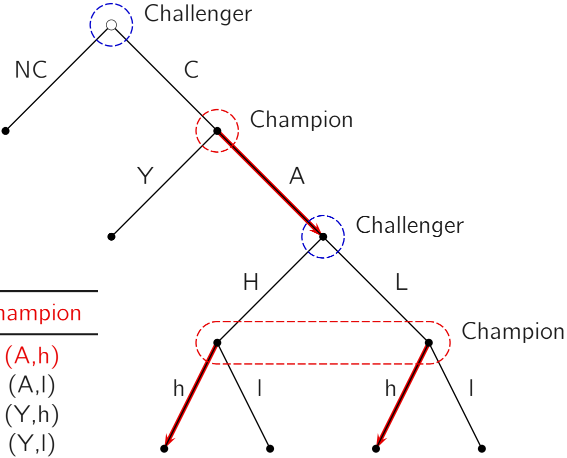
A strategy is a complete contingent plant for a player in a game

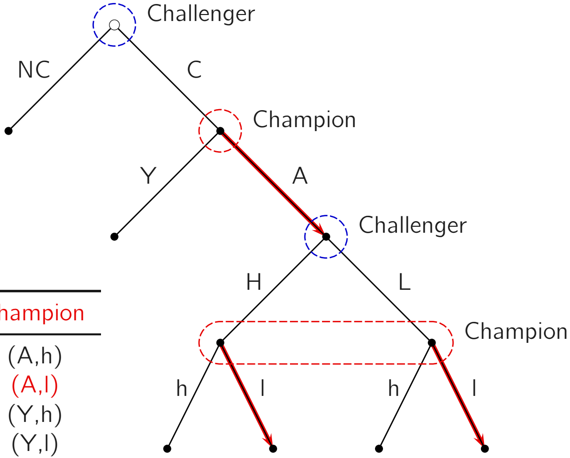
- Strategies specify a choice at every possible decision point, that is, at every information set
- "every decision point" means every decision point, even those that will not be reached (!)
- A strategy is a complete instruction manual/computer program
  - A machine would know what to do under every possible contingency
  - Even if something unexpected happens

example – heavyweight championship



challenger	champion
(C,H)	(A,h)
(C,L)	(A,l)
(NC,H)	(Y,h)
(NC,L)	(Y,l)

example – heavyweight championship



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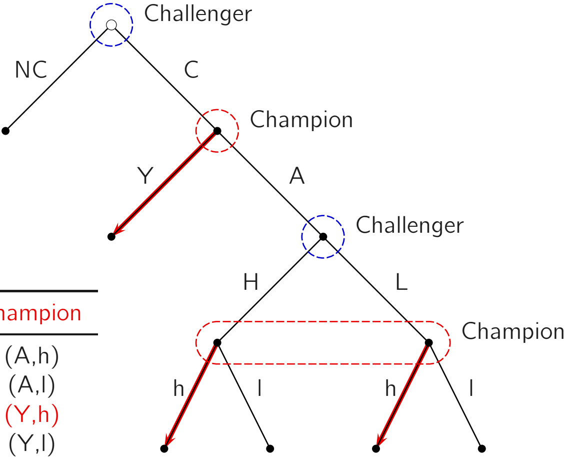
example – heavyweight championship



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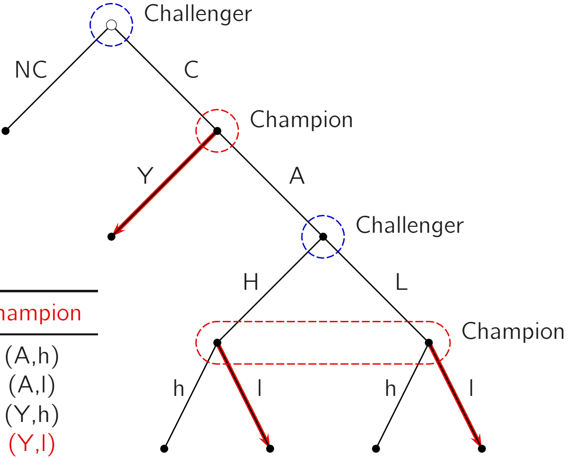
example – heavyweight championship



challenger
(C,H)
(C,L)
(NC,H)
(NC,L)

champion
(A,h)
(A,l)
(Y,h)
(Y,l)

- Any possible way of playing the game can be captured by a strategy
- Knowing each player's strategy uniquely determines an outcome
- Is knowing strategies and payoffs sufficient to analyze the situation?

A strategic form game is a mathematical object that specifies

1. The set of players
2. The set of strategies available to each player
3. A function assigning a payoff to each player for each strategy profile

example – heavyweight championship

challenger \ champion	(A,h)	(A,l)	(Y,h)	(Y,l)
(C,H)	-1 , 4	-3 , 4	3 , -2	3 , -2
(C,L)	4 , -3	5 , -3	3 , -2	3 , -2
(NC,H)	0 , 0	0 , 0	0 , 0	0 , 0
(NC,L)	0 , 0	0 , 0	0 , 0	0 , 0

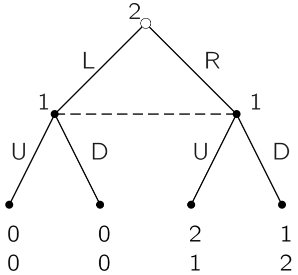
strategic vs. extensive form

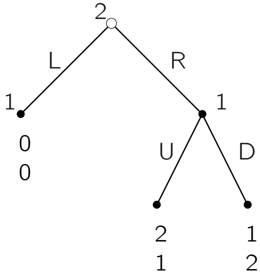
- Strategic form game often interpreted as a simultaneous move game of choosing strategies
- Choices are made independently and simultaneously

- Extensive forms are more detailed descriptions
- Strategic forms drop some information. Is this information important?
- Some people argue that strategic form games contain all the *strategically relevant* information

- An extensive form game admits a unique strategic form representation
- A strategic form game represents different extensive form games

Example: Equivalent representations





	L	R
U	0 , 0	2 , 1
D	0 , 0	1 , 2

- i denotes a generic player
- $-i$ denote the set of i's opponents
- S_i denotes the set of strategies available for player i
- Typical strategies are denoted by s_i
- $S = \times_i S_i$ denotes the set of strategy profiles – vectors that specify a strategy for each player
- s denotes a generic strategy profile
- Given $s = (s_1, s_2, \ldots, s_N)$ let $s = (s_i, s_{-i})$, where $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_{N-1}, s_N)$ is a vector that specifies a strategy for everyone except i
- $u_i(s)$ denotes the corresponding payoff for player i

A strategic form game is a mathematical object consisting of

1. A set of N players indexed by $i \in I = \{1, 2, \dots, N\}$
2. A set of strategies S_i for each player $i \in I$
3. A function $u_i : \times_i S_i \to \mathbb{R}$ for each player $i \in I$ that represents his/her payoff for each strategy profile

prisoner's dilemma

- Two suspects of a crime are arrested
- The DA has evidence to convict them for a misdemeanor (1 year in prison)
- She needs a confession for a longer sentence
- Both prisoners are offered a sentence reduction in exchange for a confession
  - If only one prisoner confesses, he walks free and his accomplice gets 5 years
  - If both prisoners confess they are sentenced to 3 years in prison each

	Keep Silent	Confess
Keep silent	$-1\,,\,-1$	$-5\,,\,0$
Confess	$0\,,\,-5$	$-3\,,\,-3$

- A "closed bag" barter is going to take place
- Each party values his object 2 and his opponent's object 3
- Each party can choose to fill the bag or not

	Full	Empty
Full	3 , 3	0 , 5
Empty	5 , 0	2 , 2

- A grimmer version https://youtube.com/watch?v=Fcno71K4v7Y

meeting in NY

- Daniel is travelling to NY to meet with Charlie
- Charlie was supposed to pick up Daniel at the train station but they forgot to specify which!
- They have no way of communicating with each other (old example?)
- They both have to choose between Grand Central Station or Penn Station

	GCS	PS
GCS	1 , 1	0 , 0
PS	0 , 0	1 , 1

battle of the sexes

- Mike and Nancy want to go on a date
- Mike wants to go to a football game while Nancy prefers the opera
- They both prefer their least preferred activity over not having a date at all

	Football	Opera
Football	5 , 1	0 , 0
Opera	0 , 0	1 , 5

- Anna and Bob simultaneously decide whether to invest in a start-up
- The start-up becomes profitable only if both invest

	Invest	Not
Invest	$2, 2$	$-1, 0$
Not	$0, -1$	$0, 0$

- Inspired by the classic film *Rebel Without a Cause (1955)*
  https://youtube.com/watch?v=u7hZ9jKrwvo
- Players drive towards each other
- They can continue driving straight or swerve to avoid a crash
- If only one player swerves he/she is a “chicken” which is something shameful but better than crashing and dying

	Continue	Swerve
Continue	0 , 0	5 , 1
Swerve	1 , 5	2 , 2

- There is a strong but slow pig and a weak but fast piglet
- They have to push a button in order to get some food
- The button is far away from the den where the food is dispensed
- Once the pig gets to the food, the piglet is pushed away and won't get to eat anything else
- The piglet only gets to eat if he gets to the food before the pig

		Fast: Press	Fast: Don't press
Strong	Press	$3, 1$	$0, 5$
	Don't press	$6, -2$	$-1, -1$

matching pennies

- Lisa and Joe secretly place a penny in their hand with either heads or tails facing up
- They reveal their pennies simultaneously
- If the pennies match, Lisa wins
- If they differ, then Joe wins

	Heads	Tails
Heads	-1 , $+1$	$+1$, -1
Tails	$+1$, -1	-1 , $+1$

	Rock	Paper	Scissors
Rock	$0\,,\ 0$	$-1\,,\ +1$	$+1\,,\ -1$
Paper	$+1\,,\ -1$	$0\,,\ 0$	$-1\,,\ +1$
Scissors	$-1\,,\ +1$	$+1\,,\ -1$	$0\,,\ 0$

uneven thumb

- Three kids simultaneously reveal a thumb pointing either up or down
- If all thumbs point in the same direction, the game ends a draw
- Otherwise, the kid with the uneven thumb looses

	Up	Down
Up	$0\ ,\ 0\ ,\ 0$	$1\ ,\ -1\ ,\ 1$
Down	$-1\ ,\ 1\ ,\ 1$	$1\ ,\ 1\ ,\ -1$

Up

	Up	Down
Up	$1\ ,\ 1\ ,\ -1$	$-1\ ,\ 1\ ,\ 1$
Down	$1\ ,\ -1\ ,\ 1$	$0\ ,\ 0\ ,\ 0$

Down

- Three firms indexed by 1, 2 and 3 sell the same commodity
- Firms simultaneously choose quantities in $[0, 100]$
- Let x be the quantity chosen by firm 1, y be the quantity chosen by firm 2 and z be the quantity chosen by firm 3
- The market price is determined by the inverse demand function

$$p(x, y, z) = 100 - x - y - z$$

- Firms have constant marginal cost equal to 2 so that profits are

$$u_1(x, y, z) = \Big(p(x, y, z) - 2\Big)x = -x^2 + (100 - y - z)x$$
$$u_2(x, y, z) = \Big(p(x, y, z) - 2\Big)y = -y^2 + (100 - x - z)y$$
$$u_3(x, y, z) = \Big(p(x, y, z) - 2\Big)z = -z^2 + (100 - x - y)z$$

- Two firms indexed by 1 and 2 sell commodities that are imperfect substitutes
- Firms choose prices in $[0, 10]$ simultaneously and independently
- Let p be the price chosen by firm 1, and q be the price chosen by firm 2
- The quantity demanded for each commodity depends on both prices

$$D_1(p, q) = 10 - p + \frac{1}{2}q \qquad D_2(p, q) = 10 - q + \frac{1}{2}p$$

- Firms have constant marginal cost equal to 2 so that profits are

$$u_1(p, q) = \Big(p - 2\Big)D_1(p, q) = -p^2 + \left(12 + \frac{1}{2}q\right)p - \Big(20 + q\Big)$$
$$u_2(p, q) = \Big(q - 2\Big)D_2(p, q) = -q^2 + \left(12 + \frac{1}{2}p\right)q - \Big(20 + p\Big)$$