Rationalizability

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Reading assignments: Watson, Ch. 7 & 8

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knowledge hierarchies

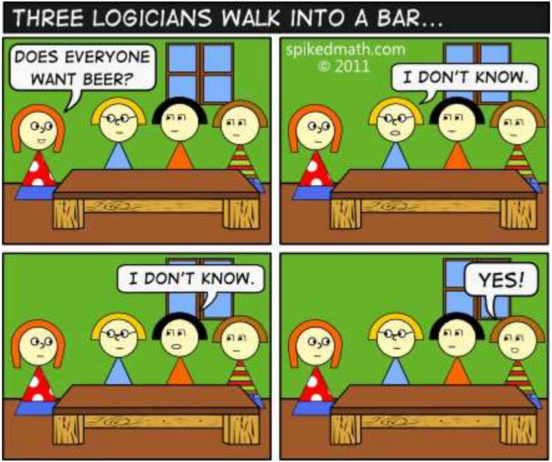
- Mutual knowledge – everyone knows
- 2^{nd} order mutual knowledge – everyone knows, and everyone knows that everyone knows
- 3^{nd} order mutual knowledge – everyone knows, everyone knows that everyone knows, and everyone knows that everyone knows that everyone knows
- ...
- Try to keep track of who knows what in the following clip https://youtube.com/watch?v=LUN2YN0bOi8

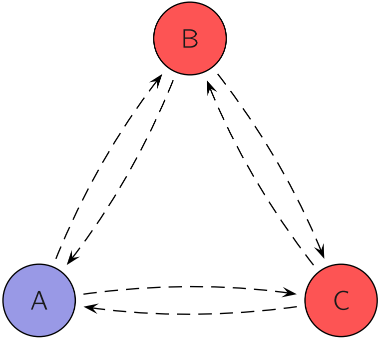
(a) A fact is mutually known if everybody knows it

(b) A fact is commonly known if everybody knows it, everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows it, and so on and so forth

David K. Lewis (1969) *Convention: A Philosophical Study*

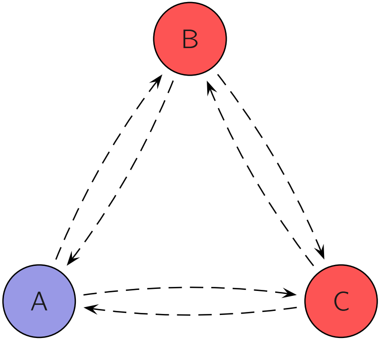
inference from knowledge





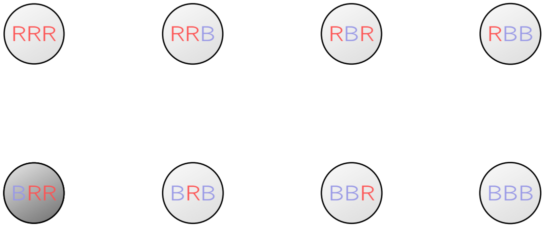
- Three logicians are wearing blue or red hats
- They can see each other hats but not their own
- They are asked “What color is your hat?”
- One by one they reply “I don’t know”

three hats



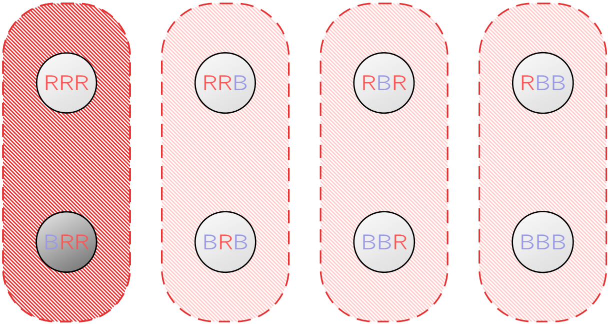
- They are told “One of you has a red hat”, something they already knew
- They are asked again “What color is your hat?”
- Ana says “I don’t know”
- Bob says “I don’t know”
- Charlie says “My hat is red”

three hats

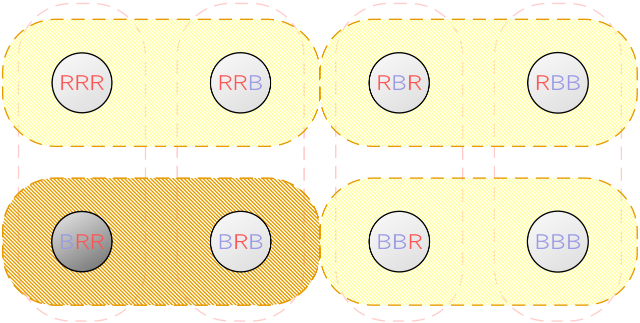


There are 8 possible configurations of hat colors

three hats

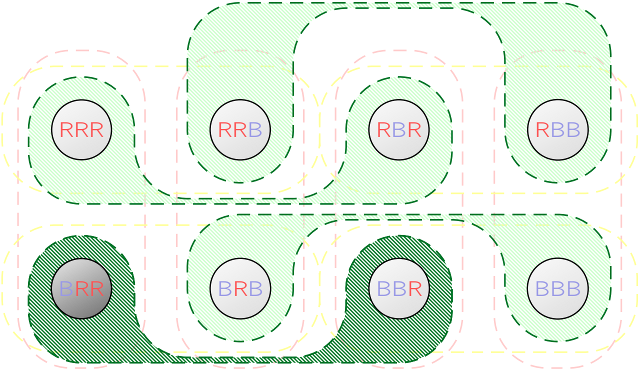


Anna's information doesn't reveal her hat color

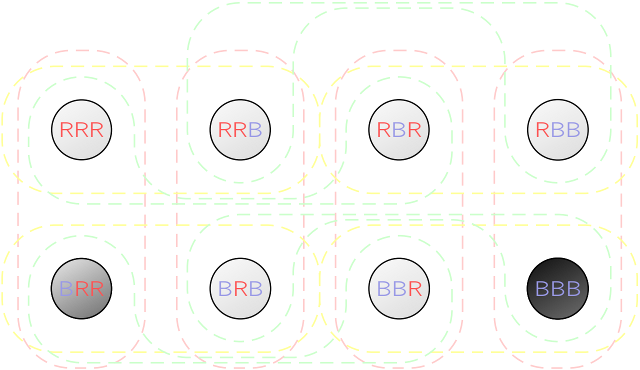


Charlie's information doesn't reveal his hat color

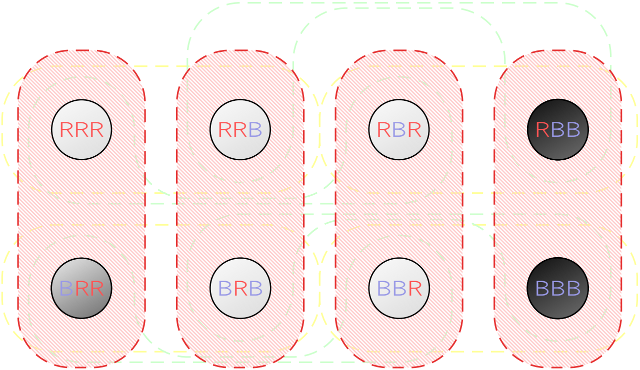
three hats



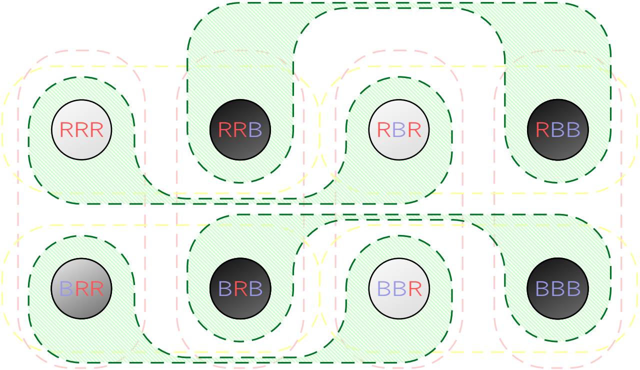
Bob's information doesn't reveal her hat color



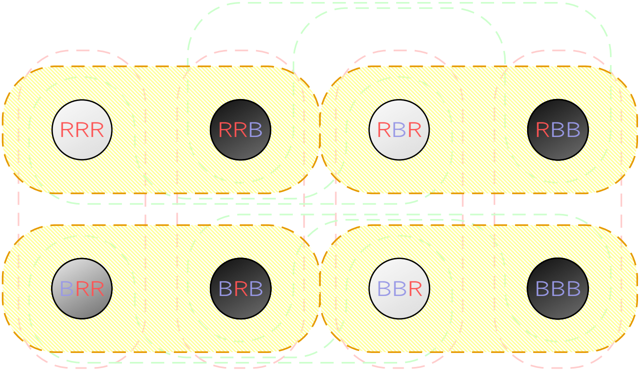
After the announcement the state BBB is no longer possible



Since Anna did not guess the color, RBB is not possible



Since Bob did not guess the color, RRB and BRB are not possible



At this point Charlie knows that BRR is the only possible state

It is commonly known that Charlie is the only one who knows the true state

common knowledge of rationality

- If all players are rational
  - ⇒ they choose best responses to *arbitrary* beliefs
- If all players know that all players are rational
  - ⇒ players believe that their opponents will play best responses
  - ⇒ they will choose best responses to best responses
- If all players know that all players know that all players are rational
  - ⇒ players believe that their opponents will play best responses to best responses
  - ⇒ they will choose best responses to best responses to best responses

  ...

- There is common knowledge of rationality
  - ⇒ players will choose rationalizable strategies

- Rationalizability = rationality + common knowledge of rationality
- Assuming common knowledge of rationality restricts beliefs to those that can be justified by complete arguments
- A strategy is rationalizable if it is a best response to such beliefs
- How do we determine which strategies are rationalizable?

iterated dominance

- Recall that a strategy is a best response if and only if it is undominated

- Rationality implies that players never play dominated strategies
- Rationality ⇒ we can eliminate these strategies from consideration

- Strategies that where not dominated in the original game can be dominated in the new game
- For example, strategies that were only best responses to eliminated strategies
- MK of rationality ⇒ we can eliminate these strategies from consideration

- CK of rationality ⇒ we can iterate this procedure until there are no more strategies to eliminate

common knowledge of rationality

- If all players are rational
  - ⇒ they choose best responses to *arbitrary* beliefs
- If all players know that all players are rational
  - ⇒ players believe that their opponents will play best responses
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  - ⇒ players believe that their opponents will play best responses to best responses
  - ⇒ they will choose best responses to best responses to best responses

  ...

- There is common knowledge of rationality
  - ⇒ players will choose rationalizable strategies

A strategy is rationalizable if and only if it survives the iterated removal of strictly dominated strategies

- All rationalizable strategies are best responses to rationalizable strategies
- The set of rationalizable strategies is the largest set with this property
- Hence, strategy is rationalizable if and only if it has a complete justification that takes into consideration the rationality of all players

example – A 4×4 game

	a	b	c	d
w	5 , 6	4 , 4	6 , 5	12 , 2
x	3 , 7	8 , 7	6 , 8	10 , 6
y	2 , 10	7 , 11	4 , 6	13 , 5
z	6 , 4	5 , 9	4 , 10	10 , 9

d is strictly dominated by *c*

example – A 4×4 game

	a	b	c	d
w	5 , 6	4 , 4	6 , 5	12 , 2
x	3 , 7	8 , 7	6 , 8	10 , 6
y	2 , 10	7 , 11	4 , 6	13 , 5
z	6 , 4	5 , 9	4 , 10	10 , 9

after d is eliminated, y is strictly dominated by x

example – A 4×4 game

	a	b	c	d
w	5 , 6	4 , 4	6 , 5	12 , 2
x	3 , 7	8 , 7	6 , 8	10 , 6
y	2 , 10	7 , 11	4 , 6	13 , 5
z	6 , 4	5 , 9	4 , 10	10 , 9

after y is eliminated, b is strictly dominated by c

example – A 4×4 game

	a	b	c	d
w	5 , 6	4 , 4	6 , 5	12 , 2
x	3 , 7	8 , 7	6 , 8	10 , 6
y	2 , 10	7 , 11	4 , 6	13 , 5
z	6 , 4	5 , 9	4 , 10	10 , 9

Playing *w* can be justified by 1 using the following complete argument

- 1 believes that 2 will choose c
- 1 believes that 2 believes that 1 will choose z
- 1 believes that 2 believes that 1 believes that 2 will choose a
- 1 believes that 2 believes that 1 believes that 2 believes that 1 will choose w

  . . .

- Hotelling (1929)
- Henry and George are ice-cream vendors selling the same product at the same price
- They choose a location for their vending carts along the beach
- Suppose that the beach is divided into 7 uniformly spaced regions
- On each region there are 10 people that will buy ice-cream from the closest vendor (splitting evenly if the vendors are at equal distance)
- Henry and George choose their location simultaneously
- They make $1 in profits for each costumer

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

Can Henry rationalize choosing 1 or 7?

No, because 1 is strictly dominated by 2 and 7 is strictly dominated by 6.

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

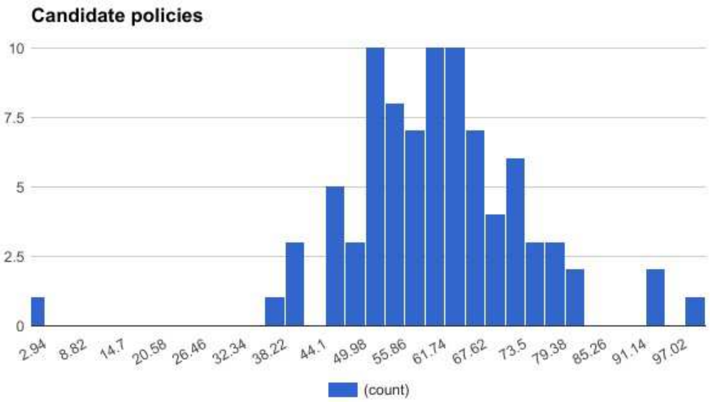
Can George rationalize choosing 2 or 6?

No, because *knowing that Henry's location will be between 2 and 6*, 2 is strictly dominated by 3 and 6 is dominated by 5

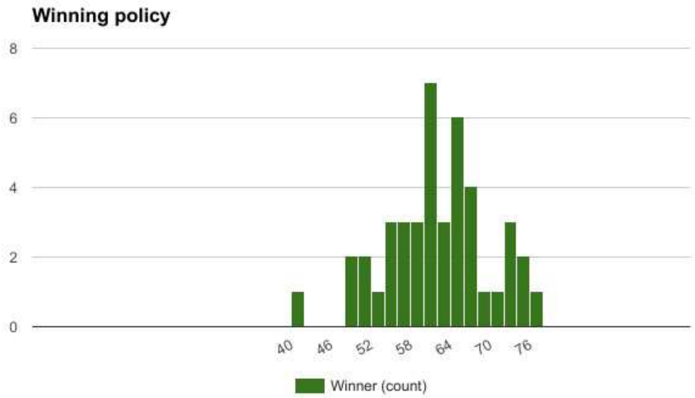
Example: location game

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

The only rationalizable strategy for either player the middle of the beach!


Candidate policies
10
7.5
5
2.5
0
2.94
8.82
14.7
20.58
26.46
32.34
38.22
44.1
49.98
55.86
61.74
67.62
73.5
79.38
85.26
91.14
97.02
(count)

median voter theorem



- Two firms 1 and 2 with constant marginal costs $c = 10$ chose quantities $q_1, q_2 \in [0, 50]$ and face prices

$$P(q_1, q_2) = 100 - q_1 - q_2$$

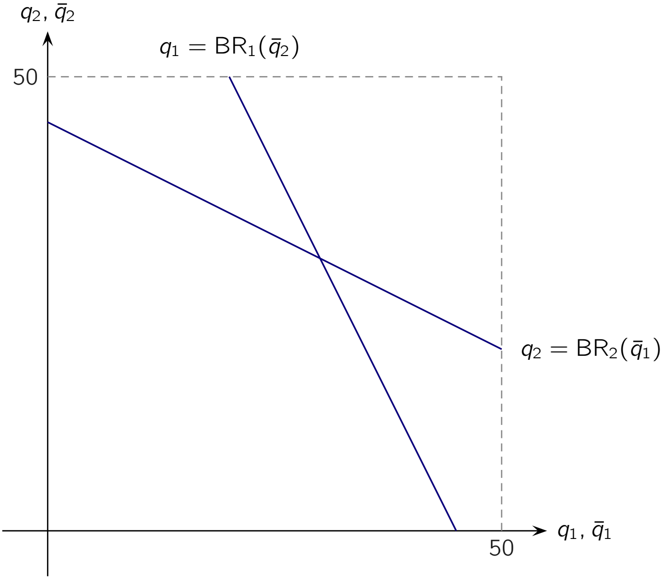
- Payoffs are given by:

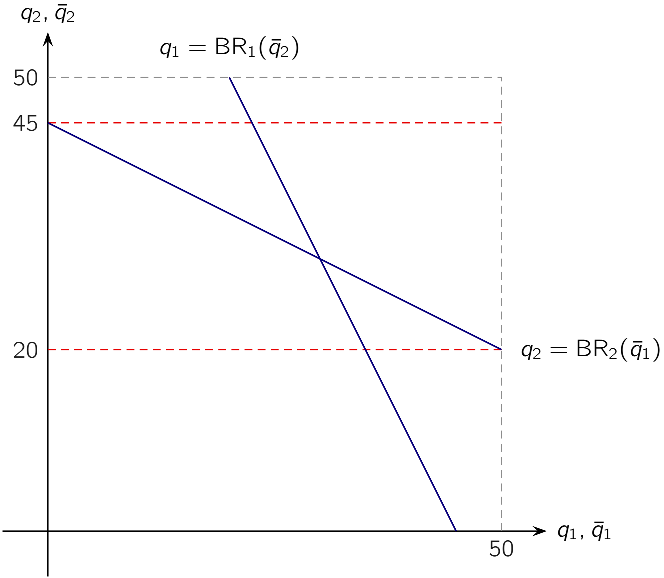
$$u_1(q_1, q_2) = (90 - q_2 - q_1)q_1 \qquad u_2(q_1, q_2) = (90 - q_1 - q_2)q_2$$

- Best responses are given by:

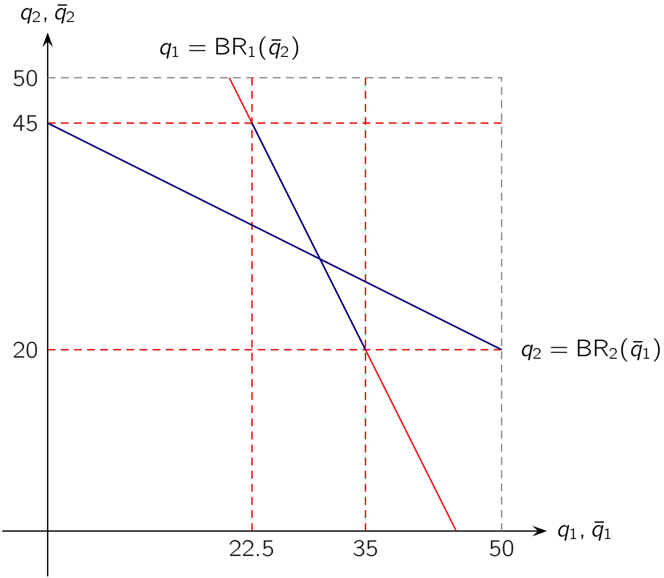
$$BR_1(\theta_2) = 45 - \frac{1}{2}\bar{q}_2 \qquad BR_2(\theta_1) = 45 - \frac{1}{2}\bar{q}_1$$

cournot competition



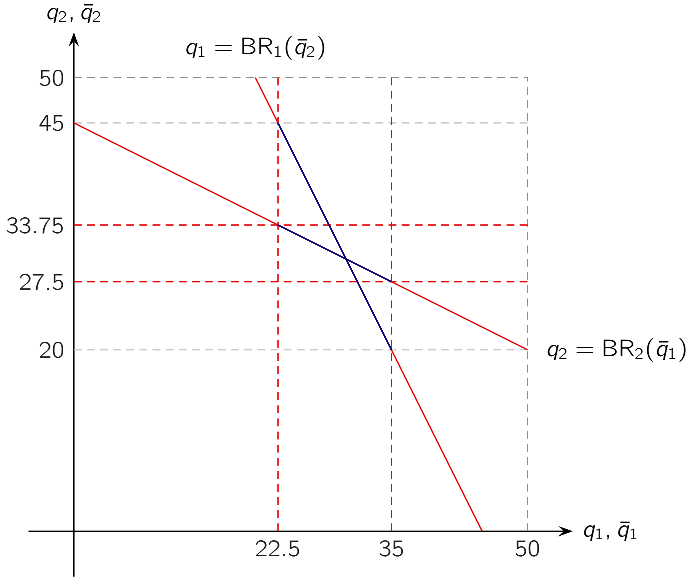


Firm 2's best response function only takes values between 20 and 45



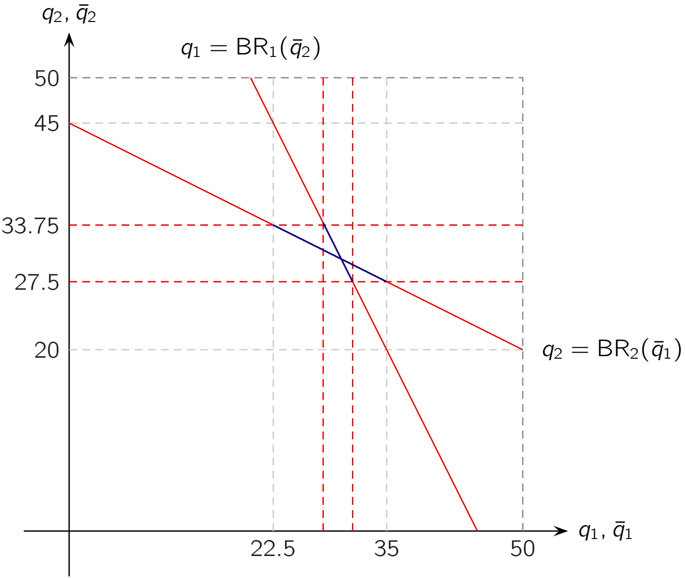
Knowing that firm 2's quantity will be between 20 and 45,

firm 1's best responses are between $\mathrm{BR}_1(20) = 35$ and $\mathrm{BR}_1(45) = 22.5$



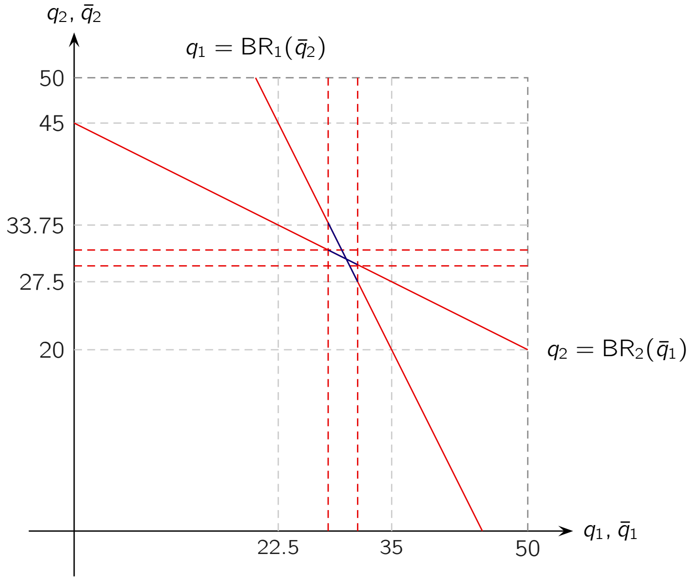
Knowing that firm 1's quantity will be between 22.5 and 35,

firm 2's best responses are between $\text{BR}_2(22.5) = 33.75$ and $\text{BR}_2(35) = 27.5$

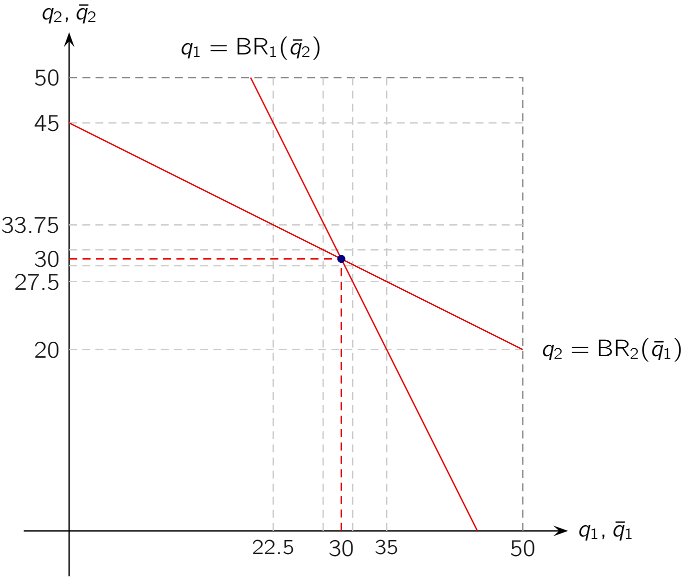


Knowing that firm 2's quantity will be between 27.5 and 33.75,

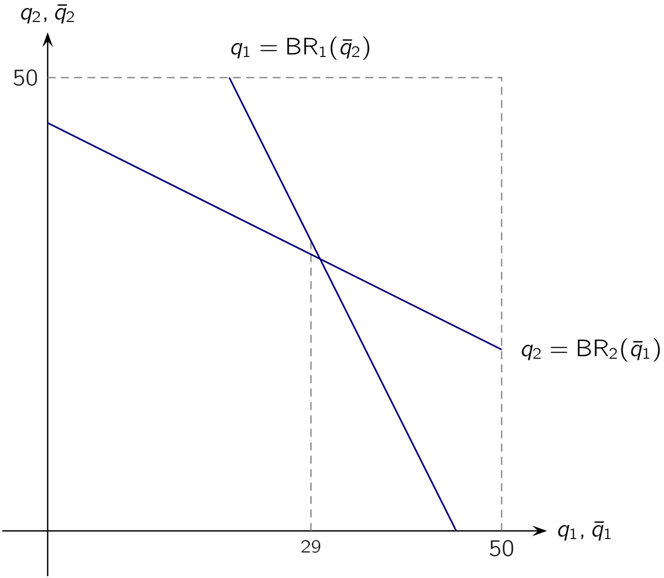
firm 1's best responses are between$\text{BR}_1(27.5) = 31.25$ and $\text{BR}_1(33.75) = 28.125$



Knowing that firm 1's quantity will be between 28.125 and 31.25, firm 2's best responses are between $\mathrm{BR}_2(28.125)$ and $\mathrm{BR}_2(31.25)$

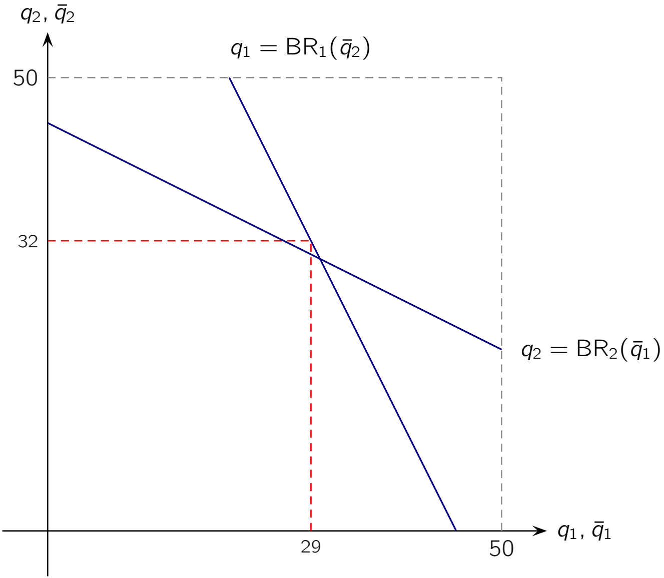


Continuing this process, the only rationalizable strategy for each firm is $q_i = 30$

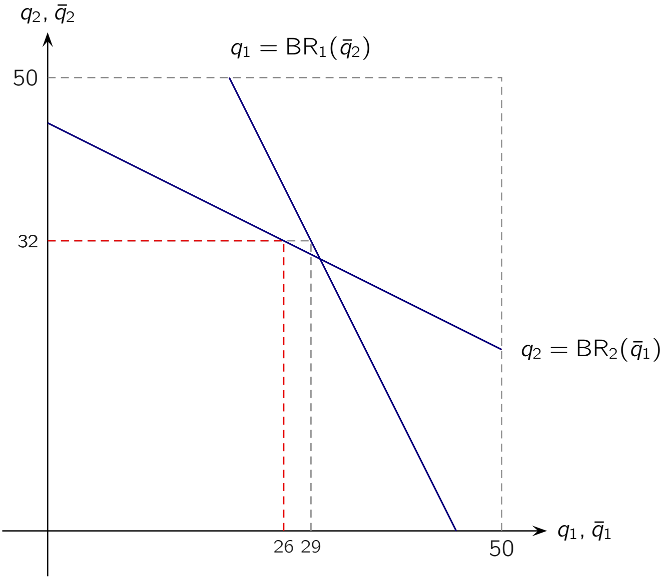


Can firm 1 rationalize choosing $q_1 = 29$?

cournot competition

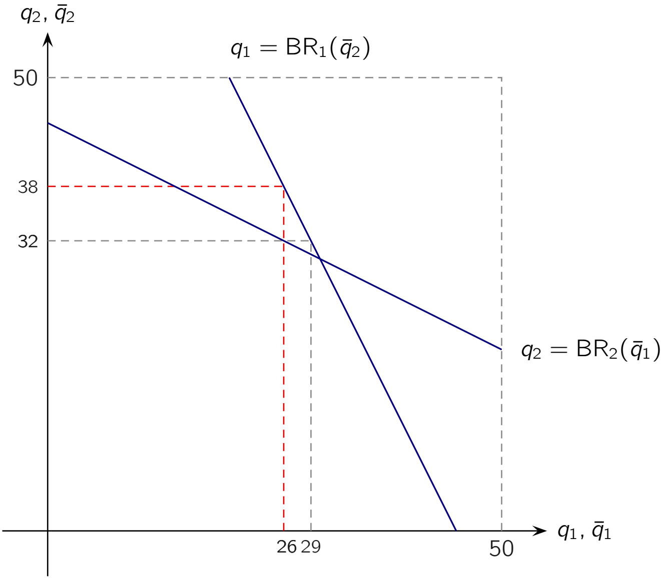


For firm 1 to choose $q_1 = 29$, it must believe that firm 2 will choose on average $\bar{q}_2 = 32$

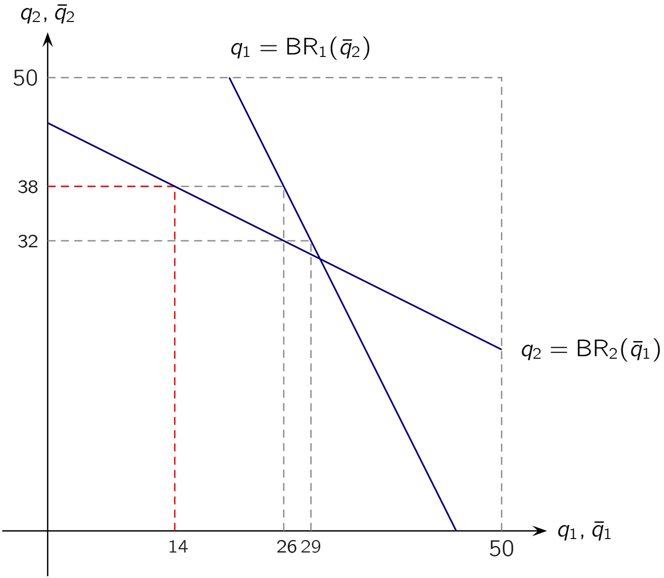


For firm 2 to choose $q_2 = 32$, it must believe that $\bar{q}_1 = 26$

cournot competition

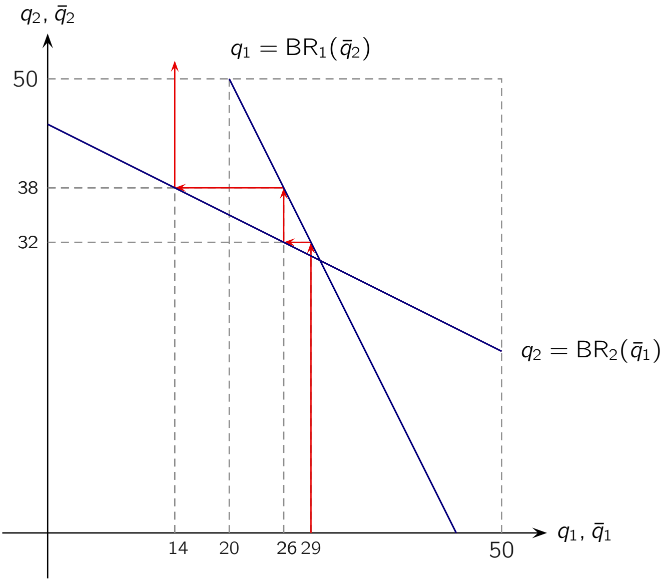


For firm 1 to choose $q_1 = 26$, it must believe that $\bar{q}_2 = 38$



For firm 2 to choose $q_2 = 38$, it must believe that $\bar{q}_1 = 14$

cournot competition



This is never rational because firm 1 will always choose $q_1 > 20$

Hence firm 1 cannot rationalize choosing $q_1 = 29$