Perfection

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Reading assignments: Watson, Ch. 15 & 19

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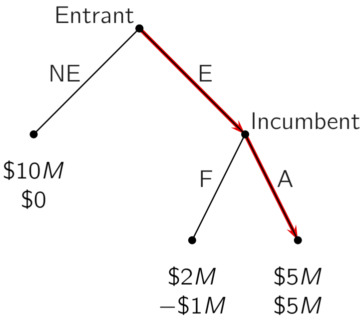
subgame perfect equilibrium

Unless someone hands me at least 500$ in cash right now,
I will fail the entire class.

- If I failed the entire class, I would lose my job and maybe worse
- If you understand this, you would not take my threat seriously
- Reasonable prediction: nobody should give me any money

- You paying up and me failing you unless you pay is in fact a NE of the strategic form game
- The dynamic structure of the game matters

entry deterrence



	E	NE
F	2 , −1	10 , 0
A	5 , 5	10 , 0

There are two Nash equilibria in pure strategies, but (F,NE) does not seem to be intuitive because, if the Entrant does enter, the Incumbent is strictly better off Accommodating

- If the game reaches the point to carry out an "incredible threat", it is not rational to do so
- This does not show up explicitly in ex-ante strategic-form analysis when looking at strategies that do not trigger them
- Reasonable under commitment, e.g., if a robot or lawyer is programmed ex-ante to play on my behalf
- Without commitment, then rationality restricts behavior at every decision node, not just at the beginning of the game
- Sequential rationality refines rationalizability and equilibrium
- In this class we look at subgame-perfect Nash equilibrium (SPNE)

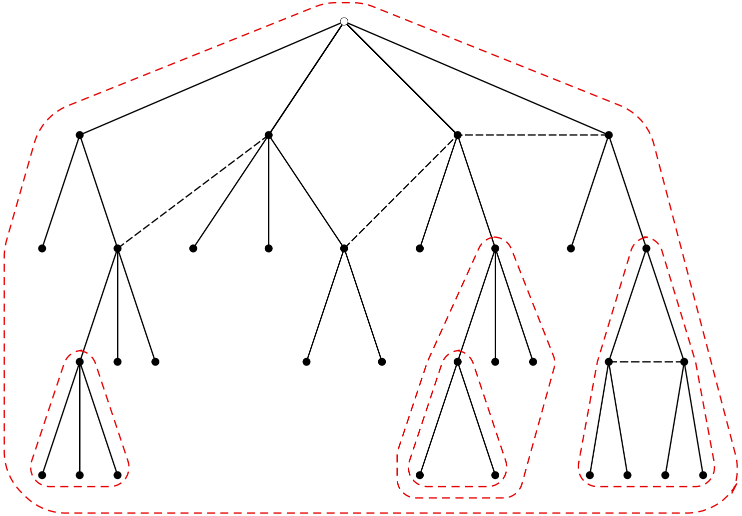
subgames

- A subgame is a part of an extensive form game that constitutes a valid extensive form game on its own

> A decision node x initiates a **subgame** if all the information sets that contain x or a successor of x contain only successors of x. The subgame initialized at x is the extensive form game conformed by x and all of its successors.

- Main requirement: not breaking information sets
- The whole game is always a subgame, other subgames are called proper
- In a perfect information game, every node initializes a subgame (why?)

example



subgame perfect equilibrium

A **subgame perfect Nash equilibrium** (SPNE) is a strategy profile that induces a NE on every subgame

- Every SPNE is a NE (why?), SPNE is thus a refinement of NE
- Simultaneous games have no proper subgames and thus NE = SPNE
- SPNE can be found using backward induction (cf. Zermelo 1913)

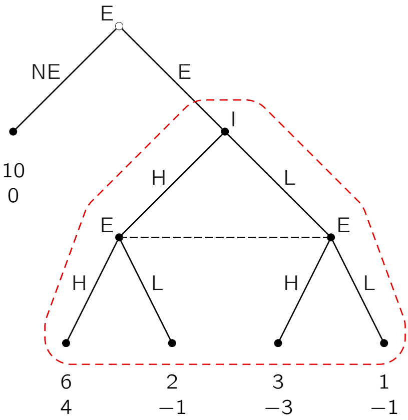
example – entrance deterrence

- Market with a single *incumbent* firm
- Potential *entrant* considers entering

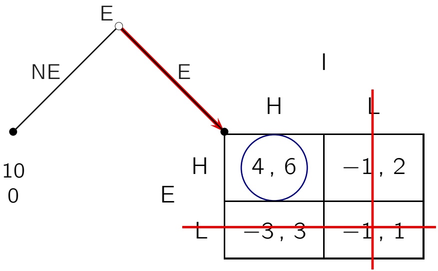
- If the entrant stays out, the incumbent makes $\$10M$ in profits
- If the entrant enters, then both firms simultaneously chose prices

		Incumbent: H	Incumbent: L
Entrant	H	4 , 6	−1 , 2
	L	−3 , 3	−1 , 1

entry deterrence



	H	L
(E,H)	4 , 6	−1 , 2
(E,L)	−3 , 3	−1 , 1
(NE,H)	0 , 10	0 , 10
(NE,L)	0 , 10	0 , 10



- Subgame perfection: if the entrant enters then both firms choose high prices
- Knowing this, the entrant prefers to enter
- $\big((E, H), H\big)$ is the only SPNE

1. Identify terminal subgames (without proper subgames)
2. Pick a NE for each terminal subgame
3. Replace each terminal subgame with a terminal node assigning NE payoffs
4. If there still are non-terminal subgames remaining, go back to step 1

- Can be multiple SPNE if subgames have multiple NE
- Under perfect information, only possible with repeated payoffs

Proposition — The strategy profiles obtained from backward induction are SPNE

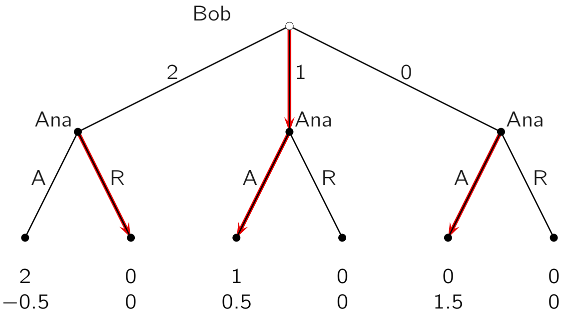
Corollary — All finite extensive form game have SPNE

bargaining

posting prices

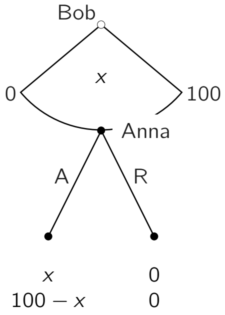
- Bob sells a mechanical
- Anna's value for the pencil is \$1.5

- Bob posts a price either \$0, \$1 or \$2
- Then Anna decides whether to accept or reject the offer



ultimatum bargaining

- Anna and Bob bargain on how to split 100$
- Anna makes a take it or leave it offer $(x, 100 - x)$ with $x \in [0, 100]$
- If Bob accepts the offer Anna takes $x$$ and Bob gets the remaining $(100 - x)$$
- If Bob rejects Anna's offer there is no agreement and they both get 0



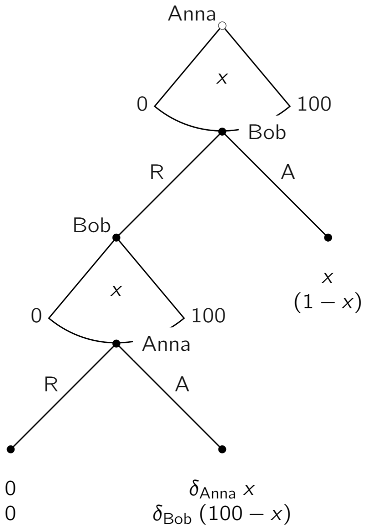
- In the unique SPNE Bob accepts any $x \leq 100$ and Anna offers $(100, 0)$

- Now suppose that Anna and Bob take turns in making offers
- In each period the proposer makes an offer $(x, 100 - x)$ and the other player decides whether to accept or to reject
- If an offer is rejected the game goes on to the following round
- Players are impatient and they discount future payoffs with discount rates $\delta_{\text{Anna}}, \delta_{\text{Bob}} \in (0, 1)$
- If the game ends with an offer $(x, 100 - x)$ being accepted at period t, the game ends with payoffs

$$\begin{aligned} u_{\text{Anna}} &= \delta_{\text{Anna}}^{t} \cdot x \\ u_{\text{Bob}} &= \delta_{\text{Bob}}^{t} \cdot (100 - x) \end{aligned}$$

- If the game ends without agreement both Anna and Bob get 0

alternate bargaining – two rounds



- Suppose that $\delta_{\text{Anna}} = \delta_{\text{Bob}} = \frac{3}{4}$
- Solve by backward induction
- **Second period:**
  - On the second period Anna will accept any offer and Bob will offer $(0, 100)$
  - If the game reached the second period

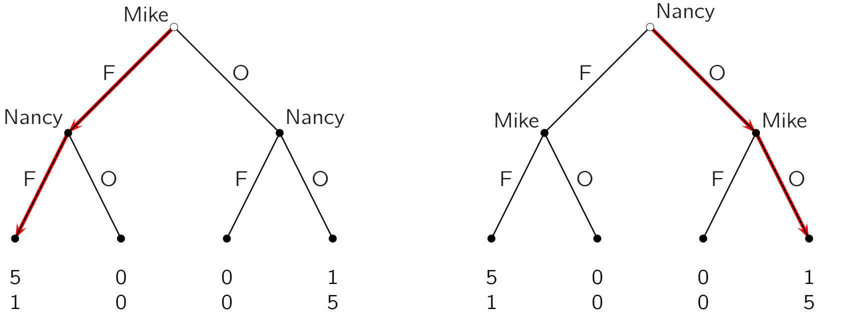
$$u_{\text{Anna}} = 0 \qquad \& \qquad u_{\text{Bob}} = \frac{3}{4} \cdot 100 = 75$$

- **First period:**
  - On the first period, Bob will accept iff $100 - x \geq 75$, i.e., $x \leq 25$
  - Anna will then offer $(25, 75)$
- The game thus will end on the first period with payoffs $(25, 75)$

sequential moves and leadership

sequential battle of the sexes

	Football	Opera
Football	5 , 1	0 , 0
Opera	0 , 0	1 , 5



- Bertrand duopoly with firms 1 and 2, constant marginal cost $c = 5$ and inverse demand

$$D_1(p_1, p_2) = 10 - p_1 + p_2 \qquad D_2(p_1, p_2) = 10 - p_2 + p_1$$

- Choices are not simultaneous
  - Firm 1 chooses its price $p_1 \geq 0$ at the beginning of the game
  - Firm 2 chooses its price $p_2 \geq 0$ after observing p_1

- Firm 1 knows that firm 2 will choose a best response

$$p_2^* = \mathrm{BR}_2(p_1) = 6 + \frac{1}{2}p_1$$

- Hence, firm 1 will choose p_1 to maximize:

$$\begin{aligned} u_1\Big(p_1, \mathrm{BR}_2(p_1)\Big) &= (p_1 - 2)\,(10 - p_1 + \mathrm{BR}_2(p_1)) \\ &= (p_1 - 2)\left(10 - p_1 + \left(6 + \frac{1}{2}p_1\right)\right) \\ &= (p_1 - 2)\left(16 - \frac{1}{2}p_1\right) = -\frac{1}{2}(p_1 - 2)\,(p_1 - 32) \end{aligned}$$

- The Stackelberg equilibrium prices are

$$p_1^S = 17 \quad \& \quad p_2^S = 14.5$$

- Profits under Stackelberg competition are:

$$u_1(p_1^S, p_2^S) = (17 - 2)\,(10 - 17 + 14.5) = 112.5$$
$$u_2(p_1^S, p_2^S) = (14.5 - 2)\,(10 - 14.5 + 17) = 156.25$$

- Under simultaneous Bertrand competition the NE is $(p_1^B, p_2^B) = (8, 8)$ and profits are

$$u_1(p_1^B, p_2^B) = (12 - 2) * (10 - 12 + 12) = 100$$
$$u_2(p_1^B, p_2^B) = (12 - 2) * (10 - 12 + 12) = 100$$

centipede game